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PUBLIC PROCUREMENT IN THE EUROPEAN UNION: THE CASE FOR NATIONAL THRESHOLD VALUES

Per Molander*

ABSTRACT. The single-most important parameter of a public procurement system is the threshold above which the framework applies. The optimization problem consists of finding a reasonable trade-off between the gains from public procurement and the administrative costs associated with procurement rules. In the present study, based on a sample of central and local government procurement operations in Sweden, an optimal threshold value in the range of 5,000–6,000 EUR is computed based on the requirement that the average gain should supersede the average cost. If a larger proportion of procurements is required to gain from the regulation imposed, a threshold value of 20,000–25,000 EUR should apply. The general conclusion is that there are strong arguments for maintaining procurement rules below the European Union threshold.

STATEMENT OF THE PROBLEM

Background

The regulatory framework pertaining to public procurement is under constant discussion. Representatives of procuring entities often criticize it for being complicated and for encumbering procurement procedures, thereby making them costly to follow and dysfunctional. Suppliers are often critical of the way the regulatory framework is applied and they sometimes put forward accusations of arbitrariness in the choice of suppliers or even corruption. It is a fact that violations occur, sometimes conspicuously so.

** Per Molander, Ph.D., is Director-General of the Swedish Social Insurance Inspectorate, Stockholm, Sweden. His main interests are in public sector management, institutional reform and welfare policy.*

A number of factors are important to the efficiency of a procurement system, such as the general level of transparency (OECD 2009), use of e-procurement (Vaidya, Sajeev, & Callender, 2006), competence of procurement officials, and organizational framework (Schotanus, Telgen, & de Boer, 2010). Among the basic design parameters, the threshold above which the regulatory framework applies is arguably one of the most important. Any design of the regulatory framework will necessarily represent a compromise between partly conflicting aims. Whatever threshold for procurement rules is chosen, there will be cases where the gains reaped from formal procurement do not cover the administrative costs. Likewise, formal procurement may not be carried out even though conditions in the situation at hand would warrant such a procedure.

The thresholds laid down in European Union (EU) public procurement directives vary depending on the domain in which they apply (for overviews, see Bovis [2005] or Arrowsmith [2009]). The dominant thresholds are 200,000 EUR for services and 5 million EUR for works. A reasonable requirement is that parameters such as these as far as possible have some empirical basis. EU procurement directives and national legislation are only to a limited extent based on analyses of potential gains and transaction costs, however; unsurprisingly, there is considerable variation below the EU threshold across otherwise similar countries (OECD, 2010). There exist a number of studies of the gains from competition, but these often refer to more fundamental reforms, such as the transition from in-house production to outsourcing preceded by formal procurement. Therefore, these studies may be difficult to use in discussions on specific aspects of public procurement regulations.

The present study was carried out in order to remedy partly this situation. A number of actual procurement operations in central and local government in Sweden were analyzed at a relatively detailed level in order to ascertain the actual gains at different levels of competition. These gains were compared with the actual transaction costs associated with more or less formalized procurement procedure alternatives. The sample is admittedly limited, but in certain respects the conclusions are so sharp that they can be expected to survive when the empirical basis is extended in future studies.

Problem Posed and Restrictions on the Solution

The basic question asked in the study is whether a national threshold below the EU threshold is justified and if so, at what level. A number of related questions have to be answered with this purpose in mind:

- Quality aspects: Many procurement operations rely on one or several quality dimensions that are somehow related to price. How should these quality dimensions be handled analytically? What is, more generally, the relationship between quality and price in actual procurement operations?
- Expected gains: What gains can be expected from a formal procurement subject to competition as compared with direct procurement not preceded by formal exposure to competition?
- Transaction costs: What are the transaction costs for the different parties involved, primarily procuring entities, but also suppliers, supervisors, and courts?
- Threshold values: Given an estimate of typical gains and costs in procurement operations, what is a reasonable threshold level for the application of formal procurement procedures?

A number of restrictions apply to the policy discussion. One threshold value is to be applied across a wide variety of sectors, procuring entities, projects, procurement officials, and so on, which necessitates a fairly stylized description of the process to be regulated. The procedure to be followed is supposed to be based on sealed bids; that is, secrecy concerning the number and identity of tenderers is maintained during the tender phase of the procurement process. Tenders are evaluated by the lowest price or most economically advantageous offer. The latter procedure may rely on a number of factors, such as technical merit, delivery date, aesthetic characteristics and price. Producers can be assumed to behave differently depending on which criterion is used, but the same threshold should apply for these two categories. Whether the same threshold should be used for all levels of government is an open question; currently, a lower value applies for central government in Sweden than for local government and other public entities, apparently more in order to confirm a higher degree of autonomy in the latter category than for economic reasons.

Earlier Work

A number of attempts have been made to estimate the benefits associated with public procurement rules. The general difficulty stems from defining a precise counterfactual situation in the absence of a procurement framework. The most common solution is to study a situation before and after some change has occurred, an approach that entails certain risks.

A comprehensive literature survey was published by the Australian Industry Commission (1996), based on more than 200 studies. Not all of the studies collected were restricted to gains from public procurement—some dealt with more comprehensive change such as partial or complete outsourcing—but the majority of the studies were restricted. The Commission reported a wide spectrum of gains, ranging from 50 percent to negative values. More than half of the studies lay in the interval between 10 and 30 percent, however, and the grand average was about 20 percent.

A more recent, less comprehensive survey by Keisler and Buehring (2009) confirmed the difficulties associated with defining a counterfactual situation. Apart from these surveys, there have been a number of studies of particular sectors and countries. Arnek (2002) provided a detailed analysis of the savings made in connection with exposing the production unit at the Swedish National Road Agency to competition, yielding an interval between 22 and 27 percent. This was a theoretical estimate, however; the realized potential turned out to be 13 percent. Duncombe and Searcy (2007) reported savings in New York school districts of the order of 4 percent from using recommended procurement practices. Ohashi (2009) estimated the effects of increased transparency in a prefecture in Japan to be up to 8 percent.

As for the EU level, Europe Economics, a London-based consultancy firm, provided two estimates of the benefits from procurement rules in the EU in studies commissioned by the EU administration (Europe Economics, 2006, 2011). In the 2006 report, the authors estimated the increase in value for money from the introduction of procurement directives in the 15 member states between 1992 and 2003 to be between 2.5 and 10 percent. In its report in 2011, Europe Economics not only estimated the general gains but also specified the relative importance of the most important

components (transparency, openness, etc.). General savings were estimated to be of the order of a few percent compared with the "Initial Estimated Total Value", which was basically what the procuring entity believed at the outset that the project in question would cost. This measures the procuring entities' ability to forecast costs rather than actual savings, however, and does not seem to be particularly useful for policy purposes.

The following section describes the evaluation problem in more precise terms and why the ideal solution, a randomized controlled trial (RCT), is not feasible. Behavioral assumptions on producers and procurement officials are described and reasons for deviating from the standard approach based on auction theory given. After some mathematical preliminaries—lognormal distributions and order statistics—we describe in detail how the threshold value has been computed based on the estimates of costs and gains from procurement. In the subsequent sections, the data and results are presented and optimal threshold values computed as a function of the restrictions imposed. We then present the results on the relationship between price and quality and conclude by summarizing the results and presenting some ramifications.

METHODOLOGICAL ISSUES

The Problem of Evaluation

Ideally, an evaluation of a regulatory framework should be based on an RCT. For several reasons, RCTs do not represent a viable alternative in the area of public procurement legislation. Firstly, legislation is national, which makes it difficult to apply different sets of rules in different parts of a country. Secondly, those who are subject to a treatment to be evaluated should preferably be ignorant of their status, which is of course impossible in this area. Finally, strategic behavior should be expected among both procuring entities and suppliers, a situation which distorts the outcome of an experiment.

When the first-best solution of an RCT is not feasible, natural experiments are often the fallback alternative (Dunning, 2012). A possibility that naturally comes to mind is to use the variation among countries concerning threshold values in order to estimate the effects on contract values for similar projects. A moment of reflection shows

that this is no more attractive as a basis for analysis than is the previous alternative. It is almost impossible to define matching projects in different countries that would be identical in all important dimensions. Further, countries differ in so many other respects that it is difficult to imagine the elimination of the effect of all relevant background variables. In summary, neither RCTs nor natural experiments seem to be feasible as a basis for deciding on an appropriate threshold value for public procurement.

The road followed in the present analysis has been to derive supply curves from the tenders presented in actual auctions and to estimate a generic supply curve in order to define the stylized situation on which the choice of threshold value should be based. Based on this stylized supply curve, a reference value for the price in the absence of a regulatory framework was derived. The gain is defined as the difference between this reference value and the minimum price among the tenders. Two methodological problems have to be solved in this context. The first is how to derive the underlying supply curve from the tenders presented. From the suppliers' point of view, procurement is a game of competition in which strategic aspects might influence the behavior of participants.

The second problem is to define a reference value assuming the supply curve has been derived. Behavior will vary across procurement situations, procuring officers, sectors, over time, and so on. These problems are tackled in the following subsections.

Producer Behavior

Any analysis of the costs and benefits of public procurement rules presupposes behavioral assumptions concerning both producers and procurement officers. The next three subsections justify the use of a non-strategic model of producer behavior, different from the standard model used in the academic literature.

Many factors affect the willingness of producers to take part in procurement competitions as well as their behavior once they decide to participate. Some of these factors are related to the general background or environment, while others to the specific project. Among background factors, the field of competence of the producer is of course basic. Size may be important, and sometimes minimum size (turnover, number of employees, etc.) is prescribed in procurement

background documents. The company may have a general policy that affects its choices in procurement matters, for instance concerning what types of projects to engage in, pricing, and cooperation. The order intake will obviously be important both for the interest to participate and for pricing. Given that any tender is presented under some degree of uncertainty, the attitude to risk may also be of importance. In general terms, large companies can afford to be more risk-prone than can their smaller competitors.

Among project-specific factors, the assessed profit from the project is obviously central to the company's decision to participate. Profit is determined by the costs and the price obtained. Costs are always uncertain, and these are normally estimated based on key numbers (cost per kilometer of road, per consultant-week, etc.). Specific conditions unknown to the tenderer may give rise to unpleasant surprises. In cases when the procuring entity bases its decision on both price and quality, the producer must also decide how different alternatives will fare depending on the quality-to-price evaluation formula. The identity of the customer may be important; some customers may be useful for reference in future marketing efforts. Finally, potential competitors affect the tender. This is also an uncertain factor given that neither the number nor the identity of other tenderers is known when the producer makes its choices.

The risk of cartels is always present. During the work on the present study, a cartel was suspected by the author and reported to the Swedish Competition Authority. This eventually resulted in a fine for the four companies involved.

The list is not exhaustive, but nonetheless offers a reasonably adequate picture of the diversity of factors involved.

Standard Model

The standard model used in the analysis of procurement is based on auction theory, which is a branch of game theory (see, for instance, Klemperer, 2004; Menezes & Monteiro, 2007; Milgrom, 2004). The auction model assumes a known number (n) of participants, denoted x_i, i = 1, 2, ..., n. Participants are assumed to be risk-neutral. The valuation v of what is offered varies by participant and is denoted v_i. The distribution of these valuations is also assumed to be known and is denoted by F(v); the corresponding

frequency function is f(v). A solution, if it exists, is further required to be symmetric, that is, participants are assumed to rely on the same basic decision function.

An equilibrium for the game thus defined exists but will in general depend on what is assumed about the valuations of participants. In the simplest case, namely independent private values (IPVs), the valuations are assumed to be independent of one another. There are also common value (CV) situations, where the values can be assumed to be the same for all participants, and affiliated value situations, which represent intermediates between IPV and CV situations. Under IPV conditions, the best strategy b*(v) is given by

$$b^*(v) = (n - 1) \int x\, f(x)\, F(x)^{(n-2)}\, dx \,/\, F(v)^{(n-1)},$$

where integration is between 0 and v. Another way of expressing this strategy is

$$b^*(v) = v - \int F(x)^{(n-1)}\, dx \,/\, F(v)^{(n-1)}.$$

It is possible to see the optimal bid as the actual valuation v minus a correction term stemming from the interaction with other bidders. It is clear to see that the optimal bid is closer to the actual valuation as the number of bidders rises and that the two will meet as the number of bidders approaches infinity.

If the assumption of risk neutrality is relaxed and replaced by risk aversion, bidding will become more aggressive and the price will be higher. Relaxing the assumption on independency of valuations complicates the analysis somewhat, but the deduction is similar.

In the standard model setting, procurement is simply assumed to be the mirror image of auctions. Higher bids correspond to lower prices and so on.

Problems with the Standard Model

Even a quick comparison of the standard model with the list of factors presented in the previous section indicates a serious mismatch. The standard model has a fairly narrow focus on the number of bidders and distribution of their valuations. Company policies and order intake are virtually absent, although the order situation may enter indirectly via the degree of risk aversion.

Even more troublesome is the fact that the assumptions made on information available to participants are unrealistic or even demonstrably false. Both the number of participants and the distribution of their valuations are assumed to be known. It may be reasonable (although not entirely unproblematic) to assume that the number of bidders is known in a classical auction, but this is conspicuously inadequate in procurement, where secrecy concerning the number and identity of tenderers is central to the procedural design. A limited number of studies–for instance, Matthews (1987), McAfee and McMillan (1987a, b), Levin and Smith (1994), Levin and Ozdenoren (2004), and De Silva, Jeitschko, and Kosmopolou (2009)--were devoted to the problem of tenders under uncertainty with respect to the number of participants. They support qualitatively the common-sense hypothesis that uncertainty will lead to more cautious behavior (i.e., lower prices in the case of procurement) but they cannot be said to be directly useful in a practical procurement situation.

The assumption that the distribution of valuations is known to participants is even more problematic. Quite apart from the epistemic problems associated with the assumption as such, the actual behavior of participants in itself proves that the assumption is wrong. If the distribution of values and number of participants were known, it would be possible (at least in principle, assuming a basic knowledge of auction theory) to compute the price level of the winning tender. It is then difficult to see why anyone should go to the trouble of presenting a tender whose price level is far above the winning level, given the cost of producing a tender. In one of the procurements analyzed, the number of tenders was 27, and the expected value for the tenderer with the highest price level was of the order of 10^{-33} EUR. Clearly, such a tender disproves the assumption of known distributions. In a more normal situation with 5 participants, no more than 3 tenders on average would in fact be submitted. In summary, the actual behavior of producers in procurement situations is incompatible with the standard model.

To these problems may be added that of estimating distributions even ex post. Laffont (1997) drew the conclusion that the outcome of such an estimation depends strongly on the assumptions made concerning non-observable distributions (see also Athey & Haile, 2005 on the problems of identifiability).

Klemperer (2002, p. 169-170) made the following general assessment of the academic literature:

> What really matters in auction design are the same issues that any industry regulator would recognize as key concerns: discouraging collusive, entry-deterring and predatory behavior. In short, good auction design is mostly good elementary economics. By contrast most of the extensive auction literature [...] is of second-order importance for practical auction design. The literature largely focuses on a fixed number of bidders who bid non-cooperatively, and it emphasizes issues such as the effects of risk-aversion, correlation of information, budget-constraints, complementarities, etc.
>
> Auction theorists have made important progress on these topics which other economic theory has benefited from, and auction theory has also been fruitfully applied in political economy, finance, law and economics, labor economics, industrial organization, etc. often in contexts not usually thought of as auctions [...]. But most of this literature is of much less use for actually designing auctions.

Alternative Description

If the standard strategic model seems to be unrealistic in its assumptions about the information available to tenderers and about their competence, the most natural alternative is to maintain traditional assumptions about rationality and profit maximization, while keeping in mind that decisions on participation, pricing, and other parameters are made under genuine uncertainty. It cannot be assumed that the number of tenderers or the distribution of their valuations is known ex ante. Although tenderers are aware of the existence of other tenderers, they are unable to adapt their behavior, given that they know neither the identity nor the preferences of their competitors. This means that a rational, profit-maximizing producer should put forward in its tender a bid that is close to its actual valuation of the project.

As a consequence, it will be assumed that producers reveal their true valuations when submitting their tenders and that a classical supply curve can thus be derived from the tenders. The number of

tenders will also affect the price level in a non-strategic setting, resulting in a lower expected price as the number of tenders increases.

Given that the distribution functions are estimated based on tenders actually presented, it is necessary to ask whether there is a risk of selection bias, that is, whether the supply function derived from actual tenders might be skewed in some direction. It is sometimes claimed that small and medium-sized enterprises are treated unfairly by the procurement framework, in which case the estimates would yield a distorted picture of the underlying distribution.

The Swedish National Financial Management Authority has investigated this problem by studying framework agreements among the approximately 50,000 suppliers to central government (ESV, 2008). The result is that there are no signs of discrimination against small and medium-sized enterprises. The dominant groups of suppliers in framework agreements in 2007 were enterprises with 10–19 or 20–49 employees. As a consequence, we have no reason to believe that the tenders presented represent a biased sample of the underlying distribution.

It should be added that the outcome of the analysis is not critically dependent on small and medium-sized enterprises' being correctly represented in proportion to their actual prevalence among enterprises in general. The procurement routine may work well even if they are not, provided that the barrier to entry is not prohibitive.

In accordance with the above arguments, the procurement operations forming the basis of this study were analyzed by using a non-strategic description. Distribution functions were estimated from the tenders for each procurement operation, as a basis for the counterfactual analysis of what would happen in the absence of a full-scale open procurement. As a test of the robustness with respect to the assumption of non-strategic behavior, consider what would be the difference between the models if one were to assume that the number of tenderers and their valuations were in fact known before the tenders are presented. According to the previously cited formula, the optimal price level in an auction is given by

$$b^*(v) = v - \int F(x)^{(n-1)}\, dx \,/\, F(v)^{(n-1)}.$$

The number v is the actual valuation, so the difference between the non-strategic and the strategic models is simply the second term on the right-hand side (with the sign reversed in the procurement situation). In the population under study, the average number of tenders was between 5 and 6. Given that this number was not known and that uncertainty would lead risk-neutral tenderers to use a higher number if they behaved strategically, it is appropriate to choose a higher number, say between 7 and 8, as the benchmark for a typical procurement operation. If the distribution function is approximated by a polynomial for v-values less than v_{min}, it turns out that the correction term amounts to 4 or 5 percent of v_{min}. Further, if we take into consideration that the most anxious tenderers can be assumed to be risk-averse, not risk-neutral as assumed in the basic strategic model, this figure should be further adjusted downwards, to perhaps 2 or 3 percent. The difference between the two models is consequently next to negligible in typical procurement situations. For deriving a reasonable threshold value, one can thus safely rely on tenders as good approximations of actual valuations and base an estimate of the supply curve on actual tenders.

It is reasonable to ask whether any demand-side effects from procuring entities need to be taken into account. In general, the answer is in the affirmative. In the present context, however, focus is on small procurement operations in the vicinity of a threshold yet to be determined. Within this category, procuring agencies are small customers that will not affect supply or demand conditions noticeably. The situation is very different in defense or large infrastructure projects, where the government is one of a few or the only buyer, and producers are likewise few. These projects are always large enough to justify open procurement, however.

Behavior among Procuring Entities

The gain estimated from formal procurement depends critically on what is assumed about the behavior among procurement officials in the absence of a regulatory framework. A threshold is an approximate solution of an optimization problem that is supposed to cover a wide spectrum of behavioral patterns among those who are entrusted with the task of procurement. Those who are critical of procurement rules tend to portray procuring officials as omniscient, impeccably objective civil servants with an unlimited amount of time

to follow markets and identify potential improvements, and almighty in the sense that their recommendations are always followed. At the other end of the spectrum, corruption exists as a permanent threat. In real life, we can expect to find anything between these extremes. A common problem among civil servants is a lack of time, which affects the possibilities of both keeping in touch with the relevant markets and finding the time necessary to carry out a full-fledged procurement when this is not mandatory. In extreme cases, the regulative framework offers an escape route, but otherwise the procurement official may feel pressure from those in charge of operations to use shortcuts to circumvent the rules. Often, an official coerced between different tasks will resort to solutions that are readily available and satisfactory although not necessarily optimal (Simon, 1997). In the case of procurement, this is tantamount to relying on suppliers that are already known and have been found to deliver satisfactorily. If this is repeated over a long time period, quasi-emotional ties may develop between the supplier and the procurement official; in fact, suppliers often support this process by maintaining the relationship, presenting gifts that are small enough not to qualify as bribes, etc. This process can be expected to affect pricing.

Further, procurement officials do not exercise full authority over procurement decisions. Sometimes, their recommendations are overruled by decision makers at higher levels.

How should this wide spectrum of behavioral patterns be rendered analytically in a model supposed to cover in principle any procurement situation? The problem is somewhat reminiscent of the classical problem of modeling agents in economic theory. The traditional starting point here has been the selfishly rational agent, who cares only about his or her own assets or profits without side glances at those of fellow human beings. This reduction cuts both ways. Actual human beings are led by altruistic preferences in some situations but by spite, jealousy, and other similarly problematic vices in other contexts. In this way, the Homo economicus of economic theory represents a neutral compromise between deviations in either direction. In Collard's (1978, p. 6) words:

> To be sure, economic man is incapable of sympathy, benevolence or love. But he is also incapable of envy, malevolence and hatred. In short, he is splendidly neutral to others. [...] Self-interest, it may therefore be argued, is a

neutral or middle assumption and certainly more attractive than envy, malice or hatred.

It is necessary to distinguish between the methodological concept of economic man and more realistic versions. The former, taking the form of expected utility-maximizing consumers or profit-maximizing producers that have full information and unlimited capacities for computation, is known to be incorrect, and in certain applications, the basic model is also refined to account for, for instance, incomplete information or bounded rationality. Nonetheless, the simple standard model is adequate for many applications.

The question is how the Homo bureaucraticus, acting in the role of public procurement officer, should be rendered in a model. At one extreme is the omniscient civil servant who is so well informed and has such moral fiber that the regulatory framework is superfluous; at the other end of the spectrum is the corrupt official who will not hesitate to take bribes in exchange for choosing inferior alternatives in any procurement operation. A reasonable and manageable analytical solution to the problem of finding a middle way corresponding to the neutrality of Homo economicus is to define the expected value of the underlying distribution as the reference point. This corresponds to a hypothetical situation in which the procuring official would choose a supplier at random from the distribution.

MATHEMATICAL PRELIMINARIES

The lognormal distribution was used extensively in this study. A stochastic variable X is said to be lognormally distributed (Λ) if its logarithm is normally distributed, or using mathematical language, $X \varepsilon \Lambda(\mu, \sigma)$ if and only if $\log X \varepsilon N(\mu, \sigma)$. Here, μ is the average and σ is the standard deviation. Lognormal distributions appear in a broad range of situations in geology, biology, ecology, economics, and reliability theory (for surveys, see Aitchison & Brown, 1957; and Crow & Shimizu, 1988). A simple argument for this ubiquitous appearance of the lognormal distribution is based on the central limit theorem; because sums of stochastic variables under general conditions will converge to the normal distribution; variables that grow with stochastically distributed growth rates should be expected to converge to the lognormal distribution. Why incomes or–as in the present study–prices in procurement operations should be lognormally distributed is not obvious, but a possible justification is

that the price of a good or a service is generated via a series of mark-ups defined as a percentage of the entry price. Fabiani et al. (2005) show that mark-up pricing is the dominant method of pricing in the EU. In that way, the final price becomes the product of a number of stochastically distributed price increases. Lognormal distributions have been used previously in the analysis of competitive bidding (see e.g. Laffont, Ossard, & Vuong, 1995; and Skitmore, Drew, & Ngai 2001).

By definition, a lognormally distributed stochastic variable has the frequency function

$$f(x; \mu, \sigma) = \{\exp[(\ln(x) - \mu)^2/2\sigma^2]\}/ \sigma x\sqrt{2\pi}, x > 0.$$

The following values can be derived:

$$\text{average} = \exp(\mu + \sigma^2/2)$$

$$\text{variance} = \exp(2\mu + \sigma^2) [\exp(\sigma^2) - 1)]$$

$$\text{coefficient of variation} = [\exp(\sigma^2) - 1)]^{½}.$$

In many situations, including procurement, the minimal value or threshold value is not zero but some positive value τ. This simply translates the whole distribution τ units to the right. The average will increase by τ units, while the standard deviation remains unchanged.

A population of suppliers is assumed to be available when public procurement is announced. The ensemble of producers can be characterized by a supply curve or distribution function F(.), showing how the supply level varies with price. A number of these potential suppliers, n, decide to take part in the competition, based on the various factors previously discussed. In any sample of size n of a stochastic variable X, there is a smallest value $X_{min}(n)$. If the distribution function of X is F(.) and the corresponding frequency function is f(.), the frequency function g(.) pertaining to X_{min} will take the form

$$g_n(x) = n [1 - F(x)]^{n-1} f(x),$$

and the expected value of X_{min} will be

$$E\{X_{min}(n)\} = n \int x [1 - F(x)]^{n-1} f(x) dx.$$

In simple situations, the frequency function of the extreme value can be expressed by using elementary functions, but in most cases numerical approximations are necessary. For example, the minimum

value for a sample of size 2 from a normal distribution can be computed analytically as

$$E\{X_{min}(2)\} = \mu - \sigma/\sqrt{\pi} = \mu - 0{,}564\,\sigma.$$

In larger samples, extreme values cannot be computed analytically (Harter, 1961). Cramér (1945, section 28.6) deduced an asymptotic formula for large n:

$$E\{X_{min}(n)\} = \mu - \sigma\,[\log(n)^{1/2} - (\log(\log(n)) + \log(4\pi) - 2C))/2\log(n)^{1/2} + O(1/\log(n)],$$

where C is Euler's constant (0.5772...).

The approximation is also reasonably good for small to moderate values of n, but convergence is slow (as 1/log(n), when n tends to infinity). By using the transformation from lognormal to normal, Bury (1975) deduced a corresponding formula for lognormal distributions. It was used in the present analysis.

COMPUTING THE THRESHOLD VALUE

Imposing a regulatory framework on public procurement implies both gains and costs. Gains can be assumed to be roughly proportional to the value of a given procurement, whereas costs consist of a start-up cost and a cost term that increases more slowly with procurement value. The optimization problem consists of finding a reasonable trade-off between the gains from public procurement – increased competition, reduced risk of corruption – and the administrative costs of following procurement rules. The problem of choosing a threshold value is equivalent to selecting a subset of all procurements for which the likelihood that the framework yields a positive pay-off to the procuring entity is sufficiently large.

Quality Aspects

It is notoriously difficult to come to grips with quality in the public procurement context. In certain sectors, such as architecture, form and design, or care for children or the elderly, it is difficult or impossible to formulate all desiderata as mandatory requirements. On the other hand, quality judgments open the door for arbitrariness and corruption.

The academic literature is relatively sparse; for a few examples, see Che (1993), Naegelen (2002), and Asker and Cantillon (2008). The Handbook on Public Procurement (Dimitri, Piga & Spagnolo, 2006) devotes one rather short chapter to quality issues. A common solution is to use some form of weighted sum of the quality and the price variable, where the relative weights reflect the importance assigned to each dimension. Procedures for trading quality off against price not only lack transparency to observers but also often seem to be incomprehensible to users themselves (Bergman & Lundberg, 2013; Keeney, 2002; Mateus, Ferreira, & Carreira, 2010). The result of the evaluation can depend on bids that are uninteresting to the outcome, which is referred to as "dependence on irrelevant alternatives"—a common phenomenon according to Bergman and Lundberg. In some procurement operations, criteria are used that would indicate that the procuring entity is prepared to pay an infinitely high price–an obvious absurdity. The source of these problems is often the relative scales of evaluation normally applied when price and quality are to be weighed together. For the quality dimension, an absolute scale is most often used, but for the price scale, the standard approach relates the prices to the least expensive alternative. If the weight assigned to quality is sufficiently high, infinitely high prices will be accepted.

Given the wide variety of methods used for trading quality off against price, the question arises of how quality dimensions should be handled in the present analysis. If the consumption functions of procuring entities were known, different tenders could be ranked according to the welfare scale implied, but they are not.

An important observation that indicates a feasible solution is that procurement operations fall into two distinctive classes: one in which quality dimensions are important, being weighted by at least 50 percent, and a second category in which quality is used only for marginal adjustments of the ranking based on price (quality weight at most 20 percent).

The criterion suggested as a solution for the former category is to use the ratio between quality and price for the alternatives under comparison. This is simple to use and is supported by the text of the relevant EU directive (Dir. 2004/18/EC, "Best Value for Money," pt. 46 of the preamble). If the procurer wishes to eliminate the risk that low-quality alternatives win the contest by offering very low prices,

restrictions on the lowest acceptable quality should be entered into the terms of reference. Such restrictions, if used, should not affect the zero level of the quality scale used by the procuring entity.

For the second category, where the procuring entity has signaled that price is far more important than quality, ranking on price alone has been used. It has been verified that the ranking of alternatives in these cases was not altered as a result of this modification.

In line with this criterion, the independent variable used in the present study for ranking suppliers and forming supply curves when quality aspects are important in the evaluation is the price per quality unit. When quality is less important, the independent variable is price alone, as in cases where only price was used in the original evaluation.

Gains

Recall that the expected value over the population of potential tenders is taken as the reference price in the present study. This can be thought of as the outcome that would result if the procuring entity picked a producer at random among those prepared to deliver the goods or services demanded.

Following this definition, the gain from an actual procurement operation is computed in the following way. Based on the tenders submitted, a distribution function is estimated by using a lognormal function with three parameters: the zero value (τ), the average (μ), and the standard deviation (σ). The gain is the difference between the expected value and the lowest price among the tenders. If the population of procurements is normalized, the average gain will approach the gain computed from order statistics by using the average number of tenders in the population.

By using the distribution function thus derived, it is also possible to answer questions of the type, “What would happen to the price level if, instead of carrying out a full-scale procurement, the procuring entity approaches, say, three suppliers chosen at random from the population of suppliers?”

The gain from procurement computed in this way is an underestimate, for at least two reasons. Firstly, the effect of merely subjecting suppliers to potential competition is not included. If only

one tender is submitted, the gain will be zero by definition. This underestimates the real effect compared with the situation in which the procuring entity approaches a producer directly, as the following example shows. In one of the procurement operations studied in this project, an official in a municipality in southern Sweden approached a university institution in order to organize a course for its middle management. The institution offered to take care of the course for the price of 55,000 EUR (500,000 SEK; approximately 1 EUR = 9 SEK). The procurement director explained to the official that a project of that size had to be advertised. This was also carried out, and one tender was submitted–from the same institution that had offered to take care of the course, but now the price was 44,000 EUR. Subtracting the administrative cost of formal procurement, the municipality gained about 16 percent on the original price by exposing the supplier to (potential) competition.

Secondly, no attempt is made to estimate the gains from the reduced risk of cartels or corruption in different forms. The description used assumes that corrupt behavior would lead to the selection of producers at the high end of the price spectrum, but if real corruption were present, prices would of course be affected.

Costs

When administrative costs are computed, it is important that they are limited to the additional costs generated by the public procurement framework. Even a direct procurement from a supplier that is known by the procuring entity requires a detailed specification of what is to be delivered. As a consequence, the administrative cost associated with procurement rules should comprise potential extra costs because of more complicated procurement documentation, advertising, the processing of the tenders submitted, and the final announcement of the result.

It is open to discussion whether the administrative cost among suppliers should be included, representing a socioeconomic rather than a pure public finance perspective. It can be argued that producing tenders is part of normal activities in business and that, in private markets, suppliers more or less often prepare tenders that in the end yield no profit. Both alternatives were investigated in the present study.

Choice of Threshold Value

The threshold value in a procurement regulatory framework determines how large a proportion of all procurement operations will be captured by the framework and subject to competition. Because it is a standard value, there will always be two types of errors: in some procurement operations, the gains from competition will not cover the additional administrative cost incurred, whereas some procurement operations will not be subjected to competition, although the potential gain would justify doing so. In this sense, one is faced with a trade-off between two types of errors. A requirement for the threshold value in a procurement framework is that expected gains on average should cover the transaction costs incurred by procuring entities, but this would not normally be considered to be sufficient. Margins are required to ensure that transaction costs are covered in a qualified majority of cases. The ultimately optimal threshold value depends on what proportion is required in this respect. Values have been computed for two-thirds and three-quarters. As shown in the analysis below, the variation associated with the choice of this safety margin by far dominates all other sources of uncertainty.

DATA

Data for the analysis were collected from procurement operations in central and local government during 2007 and 2008. In all, the sample comprises 76 procurement operations from central government and 50 operations from a municipality. In the central government sample, all operations supported by the Agency for Support to Higher Education (VHS) are included. This agency has worked as a consultant to other central government agencies, and the sample consequently comprises goods and services from a wide variety of sectors. The local government sample was collected from the municipality of Sunne in western Sweden, a typical Swedish municipality with 13,600 inhabitants, close to the median. General descriptive statistics are given in Table 1.

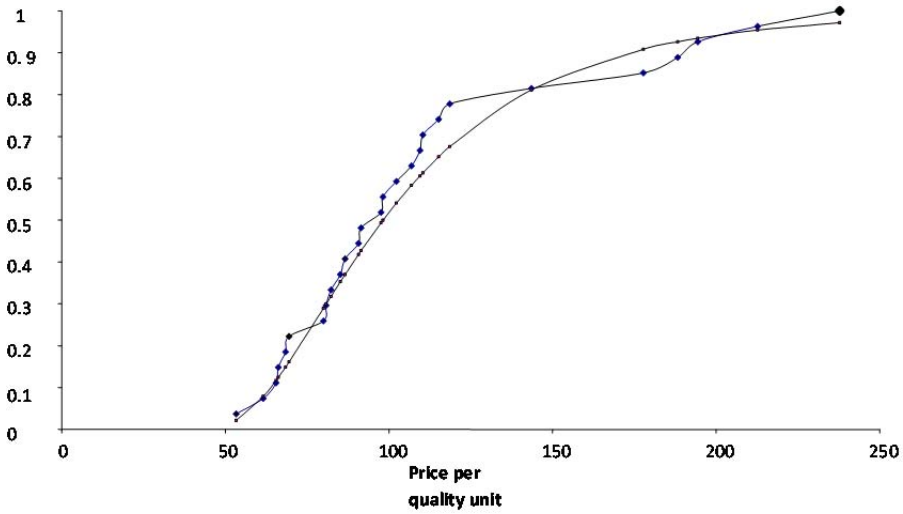
This sample may seem to be somewhat thin, but for policy purposes—determining a reasonable threshold value—only the distribution of relative gains is necessary. This distribution depends on three parameters—mean, variance, and the proportion of operations with only one tender—and for estimating these three, the sample is sufficient in size.

TABLE 1
General Statistics for the Data Sample

	Central government	Local government
Total number of operations	76	50
Average number of tenders per procurement operation	5.26	4.82
Number of operations based on price only	13	13
Number of operations with only 1 tender	14	5

An example of the distribution functions (supply curves) that have been generated from the data is shown in Figure 1. Once the distribution function has been estimated, it is possible to compute

FIGURE 1
Supply of Consultancy Services to the Swedish Agency for Public Management (27 Tenders)



Note: The figure shows observations plus a lognormal estimate of the distribution function (threshold value 55; logarithmic mean 4.147; logarithmic standard deviation 0.6125).

the expected price if, for instance, three randomly selected suppliers are invited. Given the average number of tenders in open procurement (5), it is also possible to compute the expected price for that situation. The gain, as defined previously, is simply the difference between the average price level and the expected price in the procurement situation.

RESULTS AND ANALYSIS

Gains

The relative gains from procurement as previously defined, $(EX–EX_{min})/EX$, were computed for all 126 procurement operations in the sample. The average gain in central government was 25.0 percent, and in local government, 16.9 percent. The grand average was 21.8 percent. This result agrees well with the conclusion from the previously cited literature survey by the Australian Industry Commission (1996), although the method used was different.

The gain from exposure to limited competition was also estimated by calculating the difference between the average price and the expected minimum price among three randomly selected suppliers. In this case, the expected gain (computed for central government only) was 20.3 percent.

Costs

The average cost of a procurement operation estimated by an experienced consultant in the VHS Agency was between 5 and 10 percent of the contract value. The lower value was supported by detailed information from the Agency for Public Management, as discussed in more detail below, as well as from other sources. Compared with an average gain in central government procurement of 25 percent, procurement seems to defend its position well. Naturally, there are also start-up costs for low contract values, so a break-even point has to be determined.

Detailed information on the procurement cost was supplied by a few municipalities and county councils, as follows:

- The municipality of Jönköping reported an average cost of 2,200 EUR per procurement.

- The municipality of Örebro reported a lower bound of 2,000 EUR and an average cost of 5,500 EUR.
- The county of Skåne reported costs of 6 to 7 percent of the contract value for small and medium-sized operations (50,000 to 100,000 EUR).
- The municipality of Sundbyberg reported an average cost of 10–11,000 EUR per procurement.

The large variations found among the procuring entities can be explained by differences in efficiency. The municipality of Jönköping has rationalized procurement management and it applies electronic procurement without exception. The municipality of Örebro also supplied a detailed decomposition of costs, which made the lower bound of approximately 2,000 EUR credible.

Based on the last-mentioned decomposition, it is also possible to estimate the additional cost from the procurement regulatory framework. Recall that some of the costs reported are incurred even in the absence of such a framework, given that any purchase of services or goods requires that the object to be purchased must be specified with regard to quality, quantity, time of delivery, and so on. By focusing on the simplest possible procurement, the municipality estimated that one half, that is 1,000 EUR, was a basic cost associated with direct procurement, while the other half was caused by the procurement framework.

A background report for the Public Commission on Public Procurement working in Sweden between 2010 and 2013 made similar estimates based on a broader survey (Holm, 2011). The response frequency was 36.4 percent, which raises some doubt about whether the result is biased. However, there is another, deeper source of doubt in the fact that the agencies and municipalities recognized that their answers would be used in the preparation of a new threshold value and that they therefore exaggerated their costs. Anyhow, the average additional cost emerging from this survey, computed as the difference between the cost of direct procurement and the cost of the simplified procedure, was about 1,400 EUR. In summary, these estimates were not too different from that obtained in the present limited survey, given that the cited figure pertained to projects that were on average larger and that the two surveys were carried out a few years apart.

The costs among suppliers were estimated based on a questionnaire answered by 100 companies in four sectors (IT services, facilities management, medical equipment, and general consultancy), with the number of employees ranging between 3 and 30. The financial cost ranged typically from 2,700 EUR for simple tenders to 9,300 EUR for more demanding ones. In relation to the contract value, the costs were usually about 1 percent, but they may be substantially higher if the tenderer is very interested and also feels reasonably sure of winning the contract. The argument for including supplier costs in the analysis is weak, however. Firms taking part in procurement competition regard the costs associated with tendering as part of normal business. That some expenditure items are not associated with immediate revenues is typical and firms adapt their pricing to overall costs.

Costs in the supervising agency are small in relation to the total procurement volume. The same can be said for the costs of legal procedures, given that only a small proportion of all procurements are taken to court.

Optimal Threshold Value

A basic requirement is that the expected gain from formal procurement covers the costs. Given that the expected gain is 25 percent in central government and 16.9 percent in the municipal sector, the following minimal threshold values can be deduced:

- In central government, formal procurement is justified when 25 percent exceeds 1,000 EUR, that is, when the contract value exceeds about 4,000 EUR.
- In local government, formal procurement is justified when 16.9 percent exceeds 1,000 EUR, that is, when the contract value exceeds about 6,000 EUR.
- A reasonable common threshold value for central and local governments is of the order of 5,000 EUR.

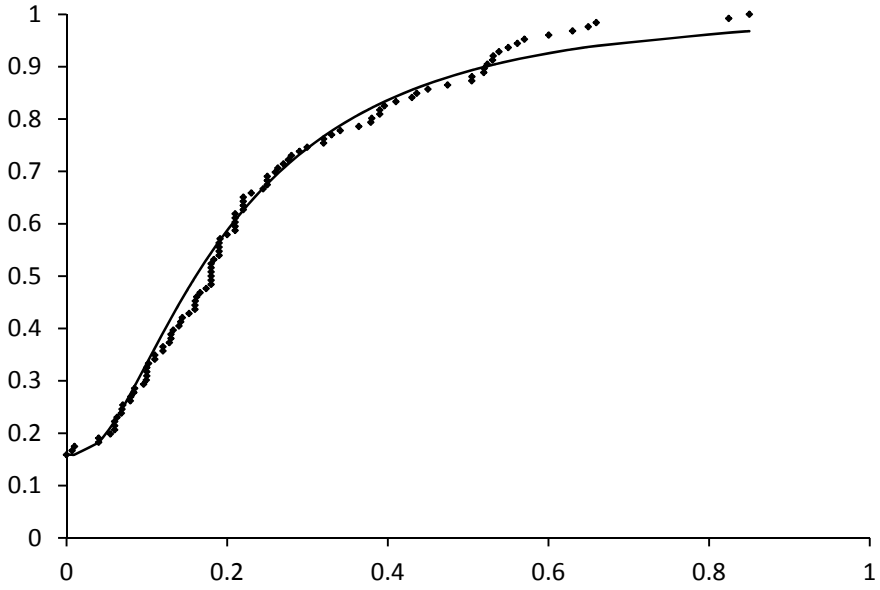
In practice, this threshold value can be considered to be too low. Given that the threshold imposes an administrative burden on the agencies concerned, one would normally require the regulatory framework to yield a value added in a larger proportion of procurements. The adequate threshold can be computed based on the distribution of the relative gains in the population under study.

This distribution is shown in Figure 2. As expected, a lognormal approximation also works well here.

Stricter requirements in this respect justify the following threshold values:

- Requiring that procurement pays in at least 50 percent of cases implies that the threshold value should be about 20 percent higher. This is equivalent to basing the design on the median rather than on the average.

FIGURE 2
Distribution of the Relative Gain from Procurement $(EX-EX_{min})/EX$ for Procurement Operations Where at Least Two Tenders Have Been Submitted



Note: The figure shows actual values plus a lognormal approximation of the distribution function; the distribution contains a point mass of 0.167 at the origin, corresponding to the operations where only one tender was submitted.

- Requiring that procurement pays in at least two-thirds of cases implies that the threshold value should be 100 percent higher.
- Requiring that procurement pays in at least three-quarters of cases implies that the threshold value should be 185 percent higher.

A 95 percent confidence interval for the average gain is obtained as the mean ± 16 percent. It is obvious that the choice of safety margin—basically a political choice—is far more important to the result than the uncertainty of the estimate.

If the cost among suppliers for producing tenders is included, the optimal threshold value is augmented by about 20 percent. Even though this could be defended as a socioeconomically based alternative, producers seem to consider the costs of producing tenders as part of normal business, as remarked above. It is worth noting that the Confederation of Swedish Enterprises in the general debate has argued for a low threshold value in order to secure a level playing field in as large a number of procurement operations as possible.

If the analysis had been based on the (unrealistic) strategic model used in mainstream studies of public procurement, the optimal threshold value would have been lower.

An Example

The trade-off between administrative costs and gains is next illustrated in some detail for one of the procurement operations in the material, namely the procurement of consultancy services by the Swedish Agency for Public Management illustrated in Figure 1. The number of tenders was 27, which is unusually high. The evaluation in this case was rather ambitious; reports from the reference projects for each of the tenderers were studied and evaluated by the staff of the procuring agency. The marginal cost of examining a new tender was 0.207 percent of the estimated value of the contract. The marginal gain from a new tender can be computed from the order statistics by using Bury's asymptotic formula. It turns out that the marginal gain equals the marginal cost when the number of tenders is 27. In other words, if the procuring agency had been able to choose the number of tenders, it would have chosen exactly the same number as was the result of the open tender procedure.

PRICE VERSUS QUALITY

More or less as a bonus, some results on the relationship between price and quality are next shown. They came out as a by-product from the computation of the price per quality unit used as the independent variable of the distribution function.

The somewhat surprising result is that among the 80-plus procurement operations where at least one quality component was used, only 20-plus exhibited a clearly positive correlation between price and quality, 20-plus a clearly negative correlation, whereas no significant correlation was found in the remaining 40 cases. Three illustrative examples are shown in Figures 3A, B, and C. The general conclusion is that no positive correlation between price and quality can be expected. Price cannot be used as a quality indicator. This is important information to procuring entities, given that large and well-established suppliers often try to use their brands as a signal of quality in order to be able to charge a higher price.

FIGURE 3A
Correlation between Price (SEK, x-axis) and Quality Score (y-axis) for Architectural Services

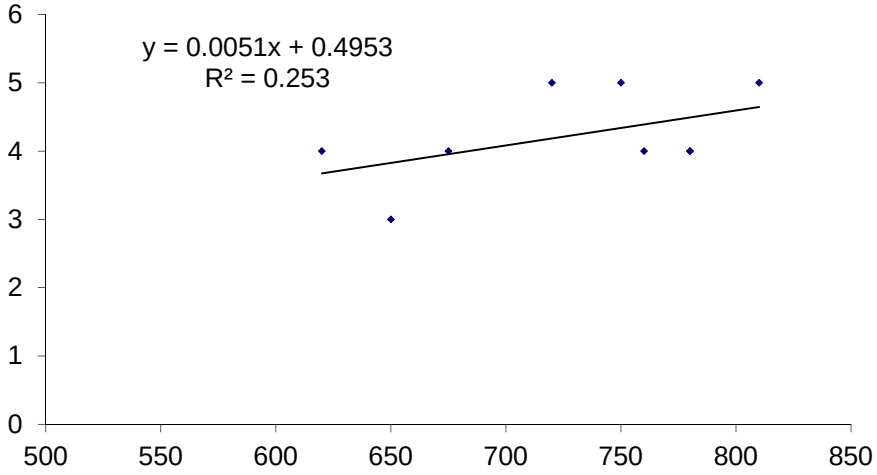


FIGURE 3B
Correlation between Price (SEK, x-axis) and Quality Score (y-axis) for IT Management

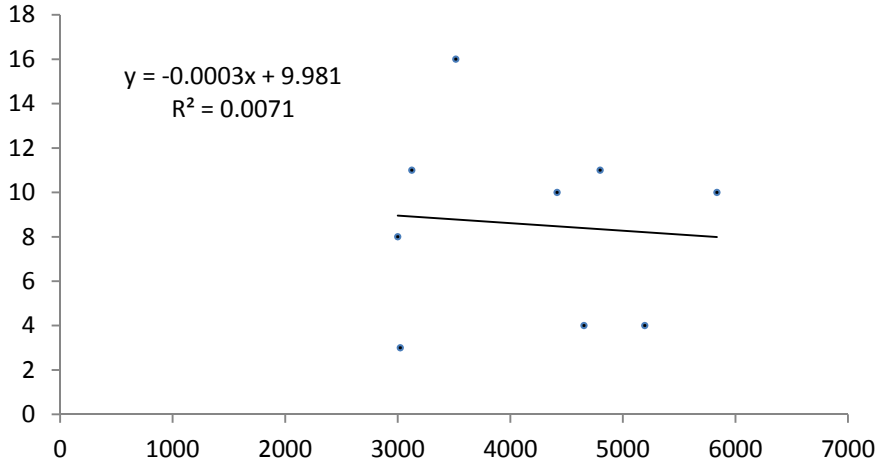
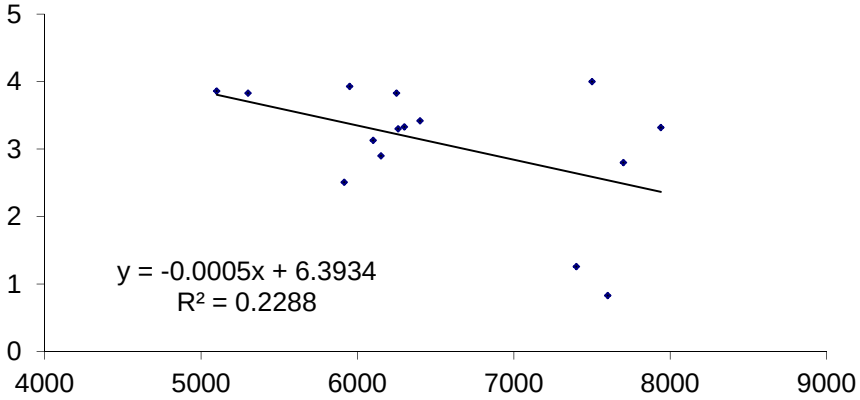


FIGURE 3C
Correlation between Price (SEK, x-axis) and Quality Score (y-axis) for PR Services



CONCLUSION

The present study, although limited in scope, sheds some light on the contentious issue of thresholds in public procurement. Among the procuring entities in Sweden–central government agencies, municipalities, and county councils–there are strong opinions about the present rules, relying on a moderate threshold level of 30 000 EUR. The regulatory framework is considered to be complex and the gains too low in general to outweigh the administrative costs. A common opinion, particularly in the municipal sector, is that Sweden should have no threshold below what is prescribed by the EU. The Confederation of Swedish Industries, by contrast, has argued for maintaining a low threshold in order to safeguard a competitive situation and to minimize the risk of corruption in various forms. Similar discussions on the pros and cons of regulating public procurement for values below the EU threshold are going on in other EU Member States.

The main results of the study are that the administrative costs associated with public procurement are normally lower than claimed in the public debate and that the gains outweigh the costs already at relatively low contract values. The critical attitude among procuring entities is understandable and serves the purpose of defending a maximum degree of freedom at the local level. Apart from this strategic argument, one reason for the distorted picture may be that one neglects the administrative costs associated with any form of procurement and forgets that only the additional cost that stems from procurement regulations should enter the calculation. Quite probably, the gains from competition are also underestimated.

The break-even point computed from the sample is about 5, 000 EUR. This is where the expected gain balances the average cost of a procurement operation. This value is in fact based on an underestimate of the gains derived from formal procurement. According to the definition chosen, no gain is registered if only one tender is submitted, although the mere condition of being exposed to potential competition will affect prices downwards–20 percent in an example from the procurements under study. On the other hand, the value is too low to be used as a general procurement threshold. Normally, one would require that the procedure yield a positive value-added in a qualified majority of all procurements. Depending on what is required in this respect, the original value should be multiplied by a

factor of 2 to 3. The general conclusion is that the current threshold value used in Sweden, 30, 000 EUR, seems to be well defended by the empirical data, even if requirements on safety margins are strong.

The method applied can also be used for computing adequate thresholds for simpler versions of procurement than the open advertised procedure. In this case, a lower threshold value is obtained, and the higher threshold value corresponding to open procurement is correspondingly adjusted upwards.

The threshold values computed depend on administrative costs and potential gains, both of which will vary from country to country. Administrative costs are determined by salaries, bureaucratic efficiency, and other variables, while potential gains will depend on typical market conditions for the sectors concerned. As a consequence, the values computed cannot be translated directly; the approach used calls for the collection of empirical data specific to each country.

As a by-product, it has been shown that the relationship between price and quality, generally speaking, is highly problematic. There is no correlation between price and quality, and price cannot be used as a quality indicator. More generally, the handling of quality aspects in public procurement would seem to merit more interest from academic researchers. There is a wide gap between what is known and what is actually used in practice, and some methods in use are liable to lead to absurdities. On the other hand, it is important that the analytic methods suggested are realistic in what they assume to be known.

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