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A COMPARATIVE STUDY OF FORMULAS FOR CHOOSING THE ECONOMICALLY MOST ADVANTAGEOUS TENDER

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ABSTRACT. Choosing the best bid is a central step in any tendering process. If the award criterion is the economically most advantageous tender (EMAT), this involves scoring bids on price and quality and ranking them. Scores are calculated using a bid evaluation formula that takes as inputs price and quality, and their respective weights. The choice of formula critically affects which bid wins. We study 38 such formulas and discuss several of their aspects, such as how much the outcome of a tender depends on which formula is being used, relative versus absolute scoring, ranking paradox, iso-utility curves, protection against a winner with an extremely high price, and how a formula reflects the weights of price and quality. Based on these analyses, we summarize the (dis)advantages and risks of certain formulas and provide associated warnings when applying certain formulas in practice.

INTRODUCTION

Procurement entails the process of obtaining works, goods and services from external suppliers, needed by the procuring entity to carry out its primary and support functions (Van Weele, 2010). Effectiveness in procurement is important for several reasons. First, every operation relies on a supply of inputs that are in many cases selected by the procurement function. Second, procurement can play a vital role in the delivery of strategic objectives. Third, efficient

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procurement can result in considerable monetary savings. Fourth, efficient procurement can help to achieve the best value for money. And fifth, when it involves public money, poor procurement decisions or failure to comply with procurement legislation can result in legal challenges (Axelsson, Rozemeiijer, & Wynstra, 2005; Van Weele, 2010; Waters, 2002).

Running a professional tendering process is particularly visible and relevant in the context of public procurement. Under European Union (EU) procurement law, objective, transparent and non-discriminatory criteria and the relative weights for quality and price have to be published in the contract notice listed on the EU *Tenders Electronic Daily* (TED). In addition, the bid evaluation methodology has to be made publicly available (e.g. Mateus, Ferreira, & Carreira, 2010).

In the current literature, various informative examples of unintended consequences of certain types of bid evaluation formulas are presented, and a number of issues related to such formulas are discussed, such as the ranking paradox (Chen, 2008; Sciancalepore, Falagario, Costantino, & Pietroforte, 2011; Smith, 2010; Sykes, 2012). In this paper, we make the following contributions to this relatively emergent public procurement literature. First, we look at some hitherto under-researched aspects of bid evaluation formulas such as protection against a winner with an extremely high price and how bid evaluation formula reflect weights of price and quality. Second, we use real tender data to evaluate the effects of using particular formulas, such as the likelihood of a ranking paradox. Third, we perform these critical assessments for 38 bid evaluation formulas collected from the literature as well as from purchasing practice. We start with a review of the relevant literature, followed by a presentation of our research methods. Subsequently, we analyse 38 bid evaluation formulas, looking at five different aspects:

- How much the outcome of a tender depends on which formula is being used,
- Likelihood of a ranking paradox,
- Shape of the iso-utility curve,
- Whether the formulas protect against winners with extremely high prices, and
- How formulas reflect weights of price and quality.

We conclude with (dis)advantages and risks of certain formulas and practical advice regarding the use of certain formulas.

REVIEW OF RELEVANT LITERATURE

Procurement is a process that can be divided into six phases: determining specifications; supplier selection; contracting; ordering; expediting, and finally follow-up and evaluation (Van Weele, 2010). This paper concerns the second phase: That of supplier selection. In this phase, qualified suppliers need to be identified, and the resulting list of qualified suppliers needs to be whittled down to the supplier (or suppliers) selected for a contract (De Boer, Labro, & Morlacchi, 2001; Wu & Barnes, 2011). The qualification stage is a sorting process, while the supplier selection stage is a ranking process (De Boer, Labro, & Morlacchi, 2001). In the qualification stage, suppliers that do not meet a minimal threshold for a certain criterion are eliminated (Aissaoui, Haouari, & Hassini, 2007). In the selection stage, buyers can rank and select suppliers based on price only, or on a combination of price and quality, the latter often being called a selection of the economically most advantageous tender (EMAT) or most economically advantageous tender (MEAT) (Bergman & Lundberg, 2013). In this paper, we are interested in approaches where suppliers bid, and the procuring entity uses a procedure based on EMAT to rank the bids.

When opting for a tendering procedure based on EMAT, the buyer needs to make a number of key decisions: Which quality (i.e., non-price) dimensions to include in the qualification stage and which in the selection stage, how to score each dimension, and how to weigh each quality dimension so as to come to one overall quality score. Attention should be given to the definition of quality; in the case of EMAT calculations in the selection stage, quality may not even be the right word. Often, most "quality" aspects are already covered in minimum requirements and do not receive a weight in the EMAT calculations. Only quality aspects on which competing suppliers offer discriminating quality are weighted and entered as "quality" in the EMAT formulas. Examples of such aspects can include environmental characteristics, technical merit or after-sales service and technical assistance (Parikka-Alhola, Nissinen, & Ekroos, 2012). For reasons of simplicity, we keep the term quality, typically the result of a weighted multi-criteria analysis resulting in one quality score for each offer.

Quality criteria are listed, prioritized by assigning weights to each criterion and these weights are usually shown to suppliers. The quality score is calculated as the weighted sum of the scores on each individual quality criterion. The minimum (maximum) quality score is 0% (100%). Because only qualified suppliers proceed from the qualification stage to the selection stage, a compensatory approach can be used for final selection, whereby a high score on one criterion (price or quality) can compensate for a low score on another criterion (Aissaoui Haouari, & Hassini, 2007). This also means that a bid with a quality level of 0% is still a valid alternative to win the tender in combination with a low enough price.

Next, the weight of quality versus price has to be decided, and finally, a choice has to be made which formula to use to combine the quality score and the price into one overall score, so that bids can be ranked (De Boer, Linthorst, Schotanus, & Telgen, 2006; Mateus, Ferreira, & Carreira, 2010). In this paper, we focus on the formula used to combine the quality score and the price of each bid into an overall score.

Earlier studies have stressed the importance of choosing an appropriate bid evaluation formula or "scoring rule" (Bergman & Lundberg, 2013; Chen, 2008; De Boer et al., 2006; Dreschler, 2009; Mateus, Ferreira, & Carreira, 2010; Pacini, 2012; Sciancalepore et al., 2011). According to Chen (2008), the bid evaluation formula plays a key role in public procurement, since it determines what 'the economically most advantageous tender' is. Chen (2008) focuses particularly on one aspect of bid evaluation formulas, namely the issue of ranking paradox. A ranking paradox refers to a situation when the original ranking of bids changes after one or more bids are added or removed. It can occur when a *relative* (as opposed to an *absolute*) bid evaluation formula is used. In the relative approach, after all bids are submitted, each bid is evaluated using a formula that takes as one of its inputs a characteristic of the total set of bids, such as the lowest quality, the highest quality, the lowest price or the highest price. Using examples, Chen (2008) shows that ranking paradox is possible for relative bid evaluation formulas because the axiom of independence of irrelevant alternatives is violated (see also De Boer et al., 2006). Pacini (2012) finds that the outcomes of certain often-used relative formulas can be manipulated by bidders, and carry with them the risk of transforming a competitive tender in a collusive tender. Chen (2008) also points out that in practice most bid evaluation formulas used are relative.

Smith (2010) claims that buyers may end up with a suboptimal outcome due to their misunderstanding of the bid evaluation formula's impact on the procurement process. He points out the significance of bid evaluation formulas in supplier selection by giving examples of tenders whose outcomes would be completely different if alternative bid evaluation formulas had been used. As an advice, he suggests, as does Chen (2008), that buyers, before using any bid evaluation formula, should perform a simulation study of its possible outcomes. This should help to determine whether the outcomes are acceptable. Sykes (2012) stresses the need to carefully assign weights to price and quality and notes that assigning different weights of price and quality for the same bid evaluation formula may yield different rankings of bids. Sciancalepore et al. (2011) present a classification of different EMAT award models and provide an example to illustrate that the choice of a particular model may also impact the outcome of a tender. Taken together, the literature acknowledges the importance of choosing an appropriate bid evaluation formula and calls for investigations into how particular formulas impact the ranking of bids.

RESEARCH METHODS

The formulas that are investigated and compared in this study were identified on the basis of an extensive review of academic and practitioner literature, an Internet search, and a review of formulas used by Negometrix, a procurement services provider, and its clients. The 38 bid evaluation formulas are listed in the Appendix. These 38 formulas vary from very simple (e.g., formula 32) to very elaborate (e.g., formula 27). Some formulas use just the weights of price and quality and each bid's price and quality score. Other formulas take information about the complete range of bids into account, such as average price, best price, or best quality. Still other formulas require the tendering entity to specify a price range on forehand (e.g., formula 26) or a reference price and/or reference quality – a benchmark to measure other offers against (e.g., formula 31). Finally, some formulas include user-defined parameters (such as formula 27), which help to tailor the formula to the subject of the tender.

We analyse these 38 formulas by simulating some of their behaviours using real tender data. Our dataset consists of 382 real tenders executed in 2011 and 2012 and collected from the tender database of Negometrix. For this study, we included all tenders which were based on EMAT, had two or more bids, were correctly awarded and archived, and had no negative or zero bids. Tendering entities (i.c. buyers) are municipalities, hospitals, universities, utility companies, water boards and other public organisations. Tendering categories are facility services and products (33.5%), temporary personnel (18.3%), project management (14.4%), medical equipment and products (16.5%), IT (6.5%), engineering (5.5%), construction (3.4%) and transportation (1.8%). Overall, we believe that the dataset provides a good representation of public procurement tenders. The breakdown of the dataset according to the official EU tender categories is as follows: Services – 57.3%; Supplies – 32.7%; and Works – 9.7%. For each tender, we know the weights of price and quality as set by the buyer, number of bids, as well as the quality score and price of each submitted bid. The quality score is obtained using weighted multi-criteria analysis and includes all relevant non-price criteria that are expressed in one number between 0% and 100%. The total number of bidders in these tenders is 1999. Some summary statistics of these 382 tenders are presented in Table 1.

TABLE 1
Descriptive Statistics of the Tenders (N = 382)

	Minimum	Median	Maximum
Number of bidders per tender	2	4	38
Weight of price	20%	50%	95%
Weight of quality	5%	50%	80%

A COMPARISON OF WINNERS AND RANKINGS

To address the question of how much of a difference it makes which formula is being used, we execute pairwise comparisons of 27 formulas. It is impossible to include all 38 formulas because 11 of them require some extra input such as reference price, price range, or user-defined parameters, and this input is not available in our dataset. We applied the 27 formulas to each tender and counted how many times each pair of formulas (a) ranked the same bid as number

1, and (b) generated the same ranking. When the bid ranked as number 1 differed between formulas, we also recorded which formula ranked a cheaper bid as number 1. We divided each count by the total number of tenders, i.e. 382, to create a similarity score. Table 2 shows for what percentage of the 382 real tenders each pair of 27 formulas ranked the same bid as number 1.

As for the bid ranked number 1, the similarity scores range from 62% to 100%. So in the majority of all tenders, all 27 formulas agree on the number 1. There are five formulas – 3, 9, 13, 32, and 35 – which relatively often lead to a number 1 that is different from all other formulas. Formulas 9 and 35 have particularly low congruence with other formulas when it comes to identifying the winning bid.

Next, we compared the same 27 formulas on the complete ranking of bids. Table 3 shows for what percentage of the 382 real tenders each pair of 27 formulas generated the exact same ranking.

As for the complete ranking, the similarity score ranges from 20% to 100%. As expected, these percentages are lower than agreeing only on the number 1 as now the entire ranking is compared. If two "outlier" formulas are ignored (formulas 9 and 35 only agree in 20%-21% of the tenders with all other formulas), we can conclude that roughly half of all formulas come up with the same rankings in 382 tenders. An exact match in ranking in all tenders is quite rare as there are only 11 exact matches out of 729 pairs. This shows that it does matter which formula is chosen: Different formulas lead to different rankings of bids.

As a third pairwise comparison, we took a closer look at those cases where the formulas disagreed on the winning bid. When a pair of formulas does not agree on the winning bid, it is of course interesting to know which formula tends to pick a lower priced bid as winner and which a higher priced bid. Table 4 shows for what percentage of the 382 real tenders the row formula ranked a cheaper bid as number 1 compared to the column formula. For each pair of formulas, the numbers above the diagonal and below the diagonal add up to the disagreement score (1 minus the similarity score of Table 2). For some pairwise comparisons, when there is disagreement about the winner, one formula always selects a bid. We conclude that it clearly does matter which formula one uses for the evaluation of bids and Table 4 helps practitioners in judging each formula for its tendency to pick a lower or higher priced winner.

RELATIVE VERSUS ABSOLUTE FORMULAS AND RANKING PARADOX

As explained before, there are two main approaches to the evaluation of bids: relative and absolute. An absolute formula does not utilize information from the submitted bids as a reference point. In other words, the score calculated using an absolute formula depends only on price and quality of a given bid. An example of an absolute formula is formula 29. Of our set of 38 formulas, 14 are absolute formulas (indicated with "A" in Table 5). A practical advantage of an absolute formula is that bidders can calculate the monetary value that buyers attribute to each weighted sub-criterion (Albano et al., 2008). This supports bidders' decisions to fulfil certain criteria or not; after all, it could cost a bidder more than the buyer's value to satisfy the criterion. This aspect is useful in guiding both buyers and bidders in preparing and submitting tenders. According to Chen (2008), another advantage of an absolute formula is that bidders can calculate their score before submitting their bid. Research also suggests that absolute formulas lead to lower bids from suppliers than relative formulas. Albano et al. (2008) suggest that predictability of the score in the case of an absolute formula might stimulate price competition.

The knowledge of the total score does not however help bidders to estimate their chances of winning the tender, as this score is only relevant in comparison with the scores of other bidders. Moreover, calculating the score is often not possible for the supplier because many quality criteria are evaluated and scored by the tendering entity only after bid submission.

When a relative formula is used, bidders can only estimate their final score as it depends on the other submitted bids, which are unknown a priori. Of our set of 38 formulas, 24 are relative formulas (indicated with "R" in Table 5). An example of a relative formula is formula 1. When a relative formula is used and one or more bids are removed or added, the original ranking of bids could change (Chen, 2008; De Boer et al., 2006), because the difference between two cheaper bid compared to the other. For example, when comparing formulas 1 and 35, they disagree on the winning bid in 33% of the 382 real tenders in our dataset; in all these cases of disagreement, formula 1 has a cheaper bid as the winning bidders' scores, derived from a relative formula, depends on one or more other bids.

While the changed ranking effect is common and known in contests (e.g. in elections or sports), it seems less intuitive when ranking bids in tendering process. This is why it has been referred to as the 'ranking paradox', a term we will also use to stay connected to other publications on the topic of procurement and bid selection, although it is not really a 'paradox'. A recent court ruling in the Netherlands stressed that it is not really a 'paradox', but simply the consequence of the tendering methodology chosen by the buyer (Court of Arnhem case 200.096.019). In a tender organized by a Dutch municipality, based on the scores calculated using a relative formula, the contract was awarded to supplier A, with suppliers B and C ranked second and third respectively. However, the buyer rejected supplier A after their first delivery, as this supplier had promised a feature their product did not have. With the two alternatives left, the buyer applied the bid evaluation formula again and now supplier C won. Supplier B disagreed and went to court, but lost the case because according to the court the buyer applied a methodology that was known to all the suppliers. Supplier B appealed, but again the court ruled that there was no problem with the methodology nor with the ranking paradox. According to the ruling (Court of Arnhem case 200.096.019), the buyer had however not been clear enough on how they would act in case one of the suppliers was excluded after announcing the results of the tender. They should have been clearer about whether they will choose the bid initially ranked number two or apply the bid evaluation formula again.

To obtain some idea about the impact of the ranking paradox in procurement, we analysed 22 of the 24 relative formulas from the set of 38. A ranking paradox is not possible for relative formula 15 (Chen, 2008). As for formula 16, it requires a user-defined parameter which is not available in our dataset and thus this formula had to be excluded from the comparison. For the sake of simplicity, we consider the case when only one bid is removed from the initial ranking. A ranking paradox can only occur in tenders with more than two bids. There are 315 of such tenders in our dataset. We applied the 22 relative formulas to the 315 tenders, and generated 22 * 315 initial rankings. Then, for each formula and each tender we removed one bid, recalculated the ranking, and compared the initial ranking with the final ranking. After the bid ranked as number 1 was removed from the initial ranking, we compared the bid ranked as number 1 in the final ranking with the bid ranked as number 2 in the initial ranking. If these two bids are different, it means that the ranking paradox has occurred. We refer to this as ‘number 1 drop-out ranking paradox’. After removing a bid ranked not as number 1 from the initial ranking, we compared the bid ranked as number 1 in the final ranking with the bid ranked as number 1 in the initial ranking. If these two bids are different, it also means that the ranking paradox has occurred. We refer to this as ‘number n drop-out ranking paradox’. The ‘number 1 drop-out ranking paradox’ and ‘number n drop-out ranking paradox’ are not the only ranking paradoxes that can occur. It is also possible that more than one bid is being removed. In practice, it obviously occurs most often that (only) the number 1 bid is removed. After all, a buyer will ‘remove’ a bid in case it turns out that number 1 does not deliver as promised. Or, a non-number 1 supplier contests the legality of the number 1 bid. There is less incentive to disqualify or contest the validity of a non-winning bid. However, the ‘number n drop-out ranking paradox’ creates the possibility of bid rigging: A non-relevant bidder submits a bid with no intention to rank number 1 in the tender, but to influence the score of other bids (for example, of a befriended bidder). With our analysis of the ‘number 1 drop-out ranking paradox’ and the ‘number n drop-out ranking paradox’ we believe we have covered the most likely cases of an already rare event of a bid being retracted after the ranking is announced.

For each of the 22 formulas, the likelihood of a ranking paradox is calculated as a ratio of the total number of cases in which a change in the ranking occurred over the total number of tenders analysed. Table 5 shows for what percentage of the 315 real tenders, either of these two ranking paradoxes occurred.

These outcomes are quite revealing in several ways. First, for only one out of 22 relative formulas (formula 5) the ‘number 1 drop-out ranking paradox’ did not occur at all, and for eight relative formulas the ‘number n drop-out ranking paradox’ did not occur. Second, for 21 formulas the ‘number 1 drop-out ranking paradox’ is more likely than the ‘number n drop-out ranking paradox’ (formula 5 is again the exception here). Third, formula 3 has relatively high chances of either of the two forms of ranking paradox occurring, while formulas 6 and 12 have the highest chances of ‘number 1 drop-out ranking paradox’ to occur. Finally, for one relative formula (formula 5) the likelihood of any of the two ranking paradoxes occurring is below 1%.

In practice, two events must happen for a ranking paradox to actually occur. First of all, a bid has to be removed (or added) after the ranking is announced. Secondly, the choice of formula and the bids submitted must be such that adding or removing a bid triggers a change in the initial ranking. Table 5 shows these probabilities of a changed ranking for different formulas. We conclude that the risk of a ranking paradox happening in practice is very low, as bids are rarely

TABLE 5
Relative versus Absolute Formulas and the Likelihood of a Ranking Paradox

	R / A*	Number 1 Rank Paradox	Number 1 Rank Paradox
1	R	1.90%	0.07%
2	R	4.15%	1.63%
3	R	5.43%	5.41%
4	R	1.27%	0.46%
5	R	0.00%	0.46%
6	R	6.35%	0.00%
7	R	2.56%	0.00%
8	R	2.54%	0.20%
9	R	0.32%	0.13%
10	R	3.49%	0.00%
11	R	1.27%	0.46%
12	R	10.16%	1.43%
13	R	3.19%	2.48%
14	R	1.59%	0.00%
15	R	0	0
16	R	N/A	N/A
17	R	4.44%	0.00%
18	R	2.24%	0.46%
19	R	2.56%	2.09%
20	R	1.27%	0.46%
21	R	1.59%	0.00%
22	R	1.92%	0.00%
23	R	4.15%	0.00%
24	R	6.09%	1.83%
25-38	A	0	0

Note: *R=Relative; A=Absolute.

retracted/disqualified in practice and our analyses show that even if they are, the probability of a change in ranking as a result is (very) low.

SHAPE OF AN ISO-UTILITY CURVE

An iso-utility curve (or: indifference curve) represents all combinations of price and quality that will receive the same score according to a formula (Chen, 2008). Its shape is not only of scientific interest; it can express different organisational buying philosophies and is of practical significance in determining the winner of a tender. Despite its importance, buyers often do not know the shape of the iso-utility curve, which is the chief reason we analyse this aspect.

To determine the shape of the iso-utility curve, we assume that the price of the cheapest offer is 10 and its quality is 20%. If a formula requires some extra input such as reference price or price range, we make an assumption to compare the shape of the iso-utility curve under the same conditions for as many formulas as possible. However, it is impossible to determine the shape of the iso-utility curves for five formulas. For formulas 2, 3, 19 and 24 this is because of division by 0 in the denominator of the price score. For formula 13 we were not able to construct the iso-utility curves because this formula does not satisfy the requirement that all points on the iso-utility curve have the same score for any set of weights of price and quality. With price indicated on the horizontal axis and quality on the vertical axis, the iso-utility curves can be straight, concave, convex, sigmoid (s-shaped) or discontinuous.

The marginal rate of substitution of quality for price is defined as the price an economic agent is willing to pay to obtain one additional unit of quality. If the iso-utility curve is straight, then the marginal rate of substitution of quality for price is constant and so every unit of quality is worth the same amount of money. If the curve is concave, then the marginal rate of substitution is increasing which means that consecutive units of quality are valued more and more. In some cases, the last percent of quality reaching to 100% seems indeed worth more than the average, so buyers may at times be attracted to such shape. If the curve is convex, then the marginal rate of substitution is decreasing which means that consecutive units of quality are less and less valued. This shape seems also convincing when budgets are limited - the first bit of quality is well appreciated, but a perfect score on quality may not seem aligned with the limited budget. Sigmoid shaped curves have also attracted interest of buyers; they do not wish to reward extreme bids, neither really low bids, nor really high bids. Bids around the average are favoured relatively more.

Besides straight, convex, concave, and sigmoid, we have also noted discontinuous iso-utility curves (formulas 3, 10, 12 and 16). Using artificial data, we list in Table 6, whether the iso-utility curves are straight, convex, concave, or sigmoid for three different sets of weights of price and quality (50-50; 60-40; and 40-60). We chose these three sets of weights because it is interesting from a practical point of view, what is the shape of the iso-utility when the weights of price and quality are equal; the weight of price is higher than the weight of quality; and the weight of price is lower than the weight of quality. For other sets of weights at either side of the 50-50 weights, the iso-utility curves will have the same shape, but a different slope.

For formula 27 the iso-utility curve can be straight, concave, or convex, depending on how the user sets the parameter n. For formulas 29 and 35, the shape varies across these three sets of weights. Since six formulas (formulas 27, 28, 32, 36, 37, 38) do not use the weights of both price and quality, we report their general shape of the iso-utility curve. It is also important to note that these analyses do not provide a general proof of the shape of the iso-utility curve.

For all formulas, except two (formulas 29 and 35), the shape of the iso-utility curve does not vary with the distribution of weights for quality and price. For formula 27, the shape depends on a parameter set by the buyer. The majority of formulas, i.e. 19 out of 33, have straight iso-utility curves. For eight formulas the iso-utility curves are concave, whereas for two formulas they are convex. Four formulas have discontinuous iso-utility curves, meaning the iso-utility curve has a break. As an illustration, we display the iso-utility curves for four different formulas with the same three sets of weights for price and quality (50-50; 60-40; 40-60) in Figure 1.

Figure 1 also shows the link between the shape of the indifference curve and protection against a winner with an extremely high price which we discuss later in the paper. Looking at convex indifference curves displayed in top right panel of Figure 1, it is clear that they have an asymptote, i.e. a straight vertical line bounding the

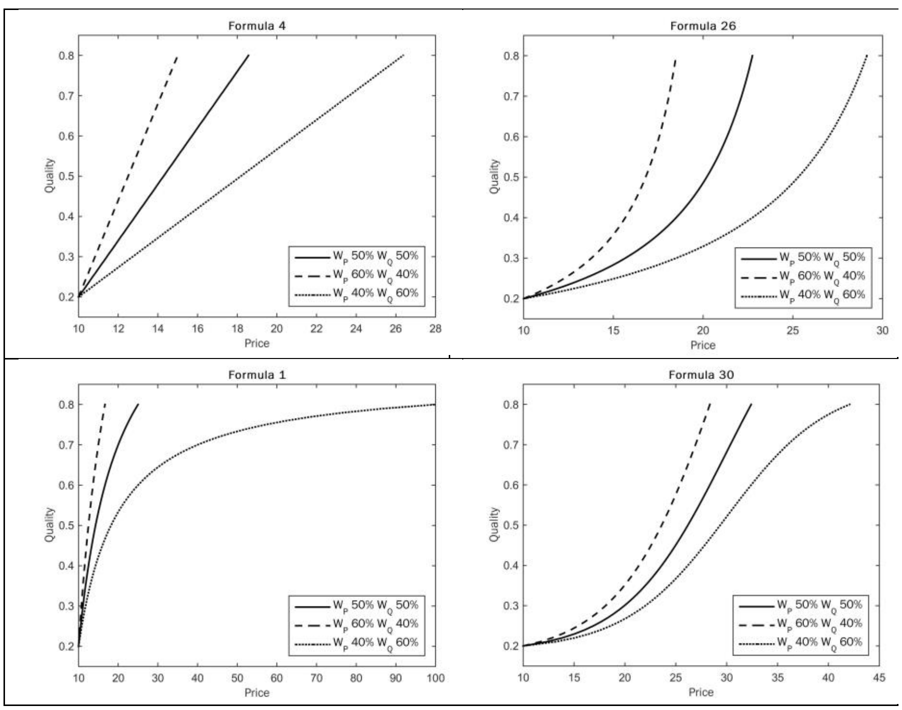
TABLE 6
Shape of the Iso-Utility Curve

Formula #	Iso-utility curve		
	(50-50)	(60-40)	(40-60)
1	Concave		
2	N/A*		
3	N/A, Discontinuous		
4-6	Straight		
7	Concave		
8	Straight		
9	Straight		
10	Straight, Discontinuous		
11	Straight		
12	Straight, Discontinuous		
13	N/A		
14	Straight		
15	Concave		
16	Concave, Discontinuous		
17	Concave		
18	Straight		
19	N/A		
20-23	Straight		
24	N/A		
25	Concave		
26	Convex		
27	Straight, Concave, Convex, depending on n		
28	Straight		
29	Straight	Convex	Concave
30	Sigmoid		
31	Convex		
32-34	Straight		
35	Straight	Convex	Concave
36	Straight		
37	Concave, when b > 0		
38	Concave, when b < 0		

Notes: * N/A = Not applicable – the curve cannot be plotted, e.g. because the shape of the curve depends on additional assumptions and/or information about actual bids.

indifference curve from the right. This means that as the quality increases, the price will never go above a certain level. Hence, formulas with convex indifference curves provide protection against a winner with an extremely high price. On the other hand, formulas with either straight or concave indifference curves do not provide protection against a winner with an extremely high price, because these indifference curves do not have an asymptote bounding them from the right.

FIGURE 1
Examples of Straight, Convex, Concave, and Sigmoid Iso-Utility Curves



An Example with a Non-straight Iso-utility Curve

To illustrate the issue with straight versus non-linear iso-utility curves, we look at a real (but simplified) example. A large public organisation intends to buy multifunctionals (machines that copy, print and scan documents). Our dataset contains several such tenders. To determine the winner, the buyer uses an often used formula 1, with a concave iso-utility curve, which is given by:

$$Score_i = \frac{P_{Best}}{P_i} W_{Price} + Q_i W_{Quality}$$

- The lowest price bid gets the maximum price score; others get fewer points pro rata.
- The quality score is the sum of the scores on the individual quality criteria times the total weight of quality.
- The total score is the sum of the price score and the quality score. The ranking is based on the sum, the highest sum ranks first. The iso-utility curve is concave, which cannot be easily seen from the formula and it is our experience that practitioners often do not know this.

The buying organisation has determined that besides price, there are three quality criteria on which the suppliers can differentiate their offers: Technical Capacity; SLA (Service Level Agreement including up-time indicators and management); and CSR (Corporate Social Responsibility).

Note that there are also a lot of minimum requirements without weight, ensuring that even if an offer has a 0% quality score, the organisation can adequately work with the contracted multifunctionals. These minimum requirements are covered in the qualification stage. In the selection stage, the buyer is looking to rank all qualified bidders on discriminating criteria. This allows the buyer to choose for a 70% weight on price and 30% on quality, and quality and price are fully compensatory. The buyer now assigns a weight to the three quality (i.e. non-price) criteria (as in most formulas[1] and most methodologies[2]). In our simplified example, the buyer has determined that Technical Capacity, SLA, and CSR are equally important; they all get 33.3% of the quality weight. Four imaginary bids are presented in Table 7, each receiving the same overall score.

Loss of Fungibility

Buyers and suppliers understand the consequences of EMAT: A score on a non-price criterion can be compensated with money.

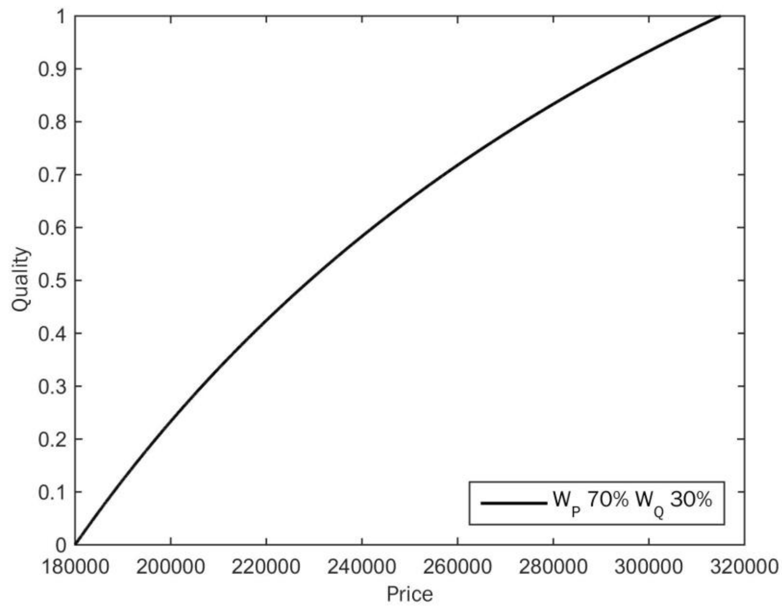
TABLE 7
Four Imaginary Bids Used in the Multifunctionals Example

Bid	A	B	C	D
Price (€)	180,000	210,000	252,000	315,000
Technical Capacity	No	Yes	Yes	Yes
SLA	No	No	Yes	Yes
CSR	No	No	No	Yes
Quality score (%)	0	33.33	66.67	100
Weighted price score	0.7	0.6	0.5	0.4
Weighted quality score	0	0.1	0.2	0.3
Total score	0.7	0.7	0.7	0.7

Suppose that the non-price criteria are fungible. Fungibility is a property stating that individual parts are capable of mutual substitution. In our example, a score on Technical Capacity can be substituted by the same score on SLA and the offer will still receive the same score. Moreover, each can be substituted by the same amount of money. In the case of bids A and B, satisfying the Technical Capacity criterion can be substituted with €30,000. For the sake of the argument, we assume suppliers score each criterion in full, or not at all.

Now we move to the case of multiple criteria. In the example, scoring a second criterion has a different monetary exchange rate. Fungibility seems to be broken. Supplier C scores in full on two criteria. With a price of €252,000, or €72,000 higher than A, he has the same utility as supplier A. The average monetary compensation for each of the two criteria he offers is €36,000. Now you could reason that his first criterion (Technical Capacity) is valued at €30,000 and his second (SLA) at €42,000 (averaging €36,000) and that he should simply understand the consequence of the concave iso-utility curve. Yet, for another supplier offering only SLA, the value of the SLA is €30,000. Fungibility seems to be broken due to the use of a formula with a non-linear iso-utility curve, as different bidders can compensate the same quality dimension with different monetary values, even though the quality weight for each dimension is exactly the same. Figure 2 depicts the iso-utility curve in our example.

FIGURE 2
Iso-Utility Curve for the Multifunctionals Example



Although we believe that the idea of fungibility of scores on quality criteria is always implicitly assumed by buyers and suppliers, fungibility is so far not demanded by public tendering law for EMAT tenders and hence we cannot argue that formulas with non-linear iso-utility curves are illegal. However, the use of formulas with non-linear iso-utility curves is contentious. For example, Nielsen (2014) argues that non-linear iso-utility could potentially violate the equal treatment principle. We suggest as a minimum, that buyers using non-linear iso-utility should be explicit about this in their documentation to suppliers.

When using a formula with a non-linear iso-utility curve, buyers should write something like: “As in any EMAT tender, scores on one quality criterion can be compensated with a similar score on another quality criterion. In addition, higher prices can be compensated with higher scores on the various weighted criteria. In this tender, the price-versus-quality compensation relationship is not linear. If your offer scores well on a certain criterion, this score influences the price-versus-quality compensation for the next criteria. In this tender, the compensation for your CSR score depends on your score on the Technical Capacity and the score on the SLA (and vice versa)."

A specific explanation depending on whether the curve is concave or convex would be even better. Such a clause will make this point clear which is otherwise hidden, hindering the transparency of the evaluation of the offers. Taking such a clause into consideration, we believe buyers do not like to run tenders in which the price-versus-quality compensation value of one criterion depends on scores of other criteria, as this seems unfair. Using formulas with a straight iso-utility curve avoids the loss of implied fungibility and such use is preferred by the authors, especially since there are so many of such formulas available.

PROTECTION AGAINST WINNERS WITH EXTREMELY HIGH PRICES

In certain situations, the ranking of a bid may become independent of its price. For example, when the weight of quality is very high and one bidder knows that he can offer a level of quality that gives him a sufficient advantage over other bidders, then he may win the tender regardless of his price. For instance, let the maximum quality score be 90 points and the maximum price score be 10 points out of 100 points. If in this situation, one bidder knows he can score more than 80 points on quality and that the quality score of the other bidders will not exceed 70 points, then he can charge anything he wants and still be ranked number 1. In other words, a formula that does not provide protection against a winner with an extremely high price is one for which under certain circumstances, the ranking of the best bid does not depend on its price. Procurement law protect buyers against 'abnormally' low prices by allowing them to ask suppliers for an explanation of an 'abnormally' low price. Various countries use the 20% lower than the average rule and there is considerable experience and jurisprudence on this aspect. Although buyers may also question 'abnormally' high prices, there is little experience in this area as 'abnormally high' priced bids do not tend to rank first. This is why it is valuable for practitioners to study this aspect of a formula.

To investigate this issue, we performed another simulation using our dataset of 382 tenders. We applied 27 formulas to each tender and generated 27*382 initial rankings. It is impossible to include all

38 formulas because eleven of them require some extra input such as reference price, price range, or user-defined parameters, and this input is not available in our dataset. Because this phenomenon is most likely to occur with a high quality weight, we set this weight to 80%, which is not unrealistic as our dataset contained tenders with this quality weight. Then, we increased the price of the top-ranked bid 50-fold. We recalculated the scores and generated 382 rankings for each price increase. We compared the initial rankings with the rankings after the price increase. If the bid ranked as number 1 in the ranking with the price increase is the same as the bid ranked as number 1 in the initial ranking, then we count this as one instance of this formula not providing protection against winners with extremely high prices. In Table 8, we report the percentage of cases where a given formula ranked the bid with the 50-fold price increase as the best bid. Formula 3 does not provide protection against a winner with an extremely high price in 75.92% of tenders, whereas the average for all other formulas is 7.93%.

TABLE 8
The Lack of Protection against an Extremely High Price, Measured as the Percentage of Cases (Out of 382 Tenders) in Which a Formula Retained the Same Winning Bid after Increasing the Price of That Bid 50-Fold

Formula #	No protection against high price	Formula #	No protection against high price
1	10.47	17	15.97
2	6.81	18	2.88
3	75.92	19	2.36
4	1.57	20	1.57
5	6.81	21	0.52
6	7.07	22	2.62
7	15.97	23	1.57
8	0.52	24	10.99
9	1.57	25	0.52
10	13.09	26-28	N/A
11	1.57	29	2.62
12	10.73	30-31	N/A
13	13.87	32	1.57
14	0.52	33-34	N/A
15	0.79	35	2.62
16	N/A	36-38	N/A

HOW WEIGHTS OF PRICE AND QUALITY ARE REFLECTED

In this section, we analyse data from four real tenders to show how formulas differ in the way they emphasize price versus quality. We selected four tenders from our dataset, each with two bidders. We selected a tender with one bid with high quality and high price, and one bid with low quality and low price. We selected three more tenders, each with two bids with similar prices: One with two low quality bids, one with two high quality bids, and one with significant differences in quality. Table 9 shows prices and quality levels (as a percentage) for all four tenders.

For 26 formulas, we calculated the "tipping point", defined as the percentage weight of price, above which the lower price-lower quality bid becomes the best bid. Note that these are bids A, C, E, and G in Table 9. As before, we cannot report the "tipping point" for eleven formulas because they require some extra input such as reference price or price range, and this input is not available in our dataset. Moreover, for formula 32 in Appendix A, the "tipping point" is not defined as the weights of price and quality do not feature in this formula. For low weights of price, starting at 0% price and 100% quality, the higher price-higher quality bid will always win. At some higher weight for price, the lower priced bid will become best bid. For example, a tipping point of 29.1% means that the higher-quality bid is ranked best bid if the weight of price is between 0% and 29.1%, and the lower-price bid is ranked best bid if the weight of price is 29.1% or higher.

TABLE 9
The Four Real-World Tenders Selected for the Analysis of How Formulas Reflect Weights of Price and Quality

	Bid	Quality	Price
Tender 1 - HiQ vs. LoQ, different P	A	0.4000	352,250
	B	0.7000	1,301,500
Tender 2 - Both LoQ, similar P	C	0.3784	140,086.34
	D	0.3818	142,065.42
Tender 3 - Both HiQ, similar P	E	0.9438	533,613
	F	0.9688	567,860
Tender 4 - HiQ vs. LoQ, similar P	G	0.2863	1.97
	H	0.7538	2.05

From Table 10, it is clear how for each formula different weights of price and quality define which bid is ranked as best bid. For example, in columns 2 and 7, some formulas rank the lower-price bid as best bid only when the weight of price is higher than 37%, while some others already do so with a price weight above 10%. In the 'low quality, low quality, same price level' tender (columns 3 and 8) some formulas rank the lower priced bid as best bid for weights of price above 1% (i.e. for nearly the entire range of possible price weights) while some others only from a price weight above 55%.

TABLE 10
Tipping Points, Defined as the Percentage Weight of Price, above Which the Lower Price-Lower Quality Bid Becomes the Best Bid (Column Numbers in the Top Row)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Formula #	HiQ vs LoQ, diff P	Both LoQ, sim P	Both HiQ, sim P	HiQ vs LoQ, sim P	Formula #	HiQ vsa LoQ, diff P	Both LoQ, sim P	Both HiQ, sim P	HiQ vs LoQ, sim P
1	29.1	19.0	29.2	92.3	16	N/A	N/A	N/A	N/A
2	23.1	0.3	2.4	31.9	17	37.0	37.9	29.9	94.1
3	23.1	0.3	2.4	31.9	18	32.2	38.5	29.5	95.8
4	20.7	18.9	28.6	92.1	19	13.0	0.2	1.2	18.9
5	29.1	19.0	29.2	92.3	20	20.7	18.9	28.6	92.1
6	29.1	19.6	29.3	92.3	21	10.0	18.8	27.9	92.0
7	37.0	38.6	29.9	94.1	22	27.2	38.4	29.2	94.0
8	N/A*	N/A*	N/A*	95.6	23	13.7	38.2	28.6	93.8
9	21.8	38.9	29.2	97.6	24	30.0	0.9	2.5	38.3
10	23.1	18.8	27.9	92.0	25	N/A*	N/A*	26.4	91.9
11	20.7	18.9	28.6	92.1	26-28	N/A	N/A	N/A	N/A
12	10.0	18.8	27.9	92.0	29	30.0	38.9	29.6	96.1
13	23.1	0.3	2.4	31.9	30-34	N/A	N/A	N/A	N/A
14	18.2	31.5	43.6	95.8	35	30.0	38.9	29.6	96.1
15	24.1	24.4	35.6	94.2	36-38	N/A	N/A	N/A	N/A

Notes: * N/A = Not applicable – the tipping point is outside the interval between 0% and 100%.

This experiment shows that some formulas have an inherent relative tendency to pick low price bids as winners. For example, let us look at the 'high quality, low quality, similar price level' tender (columns 5 and 10). In these columns, there are five formulas that seem much more sensitive to price than the other formulas. In this tender, one would expect that only a very high weight of price (>90%) would make the bidder with the much lower quality and slightly lower price win the tender. Not so for formulas 2, 3, 13, 19, and 24, since their tipping point is between 18 and 39%! These formulas incorporate not only the best (lowest) price, but also the worst (highest) price and therefore depend on a bid spread. The difference between the best and worst price defines the price evaluation range. If these best and worst price do not differ much, the formula becomes very sensitive to price. Only in the 'high quality, low quality, different price level' tender the tipping point of four of these five formulas is again comparable with the other 21 formulas. In this tender the highest and lowest price differ a lot, so the price evaluation range becomes very large making the sensitivity to price much lower. All of these five formulas show relatively low tipping points (and thus high price sensitivities) in three of the four tenders.

It is quite understandable that buyers want to include the highest quality bid and / or the lowest price bid in the EMAT formula. Buyers generally do not know exactly the highest possible quality, nor the lowest possible price before starting the tender. Bidders offering the highest quality or the lowest cost are the buyer's best approximation of relevant industry best-in-class standards. The case to include the highest price or lowest quality bid in formulas or even the average or the median seems a lot weaker, as our experiment demonstrates the unwanted side effect (loss of control) of adding such elements in an EMAT formula.

CONCLUSION

A balanced and properly functioning bid evaluation formula to choose the economically most advantageous tender is a critical task for any buyer. Listing and measuring against award criteria is an intense process getting abundant attention from both buyers and bidders, often debated and sometimes even contested in court. The formula itself often gets far less attention; formulas are often chosen without carefully analysing their properties. Our experience is that most often buyers are unaware of what alternatives exist to a formula they use.

This research is the first in its kind listing 38 different formulas used in procurement practice and analysing them on specific dimensions such as how much the outcome of a tender depends on which formula is being used, choice between relative and absolute formula, likelihood of a ranking paradox, shape of iso-utility curves associated with the formula, likelihood of not providing protection against a winner with an extremely high price and how a formula reflects weights of price and quality.

With this paper, we want to show the large variety of bid evaluation formulas that buyers can use in an EMAT tender procedure. Despite the efforts expended to identify such formulas from academic publications, websites and practitioner contacts, we are sure that the 38 formulas described here are not all that are out there. Our analyses with data from real tenders show that while each formula is unique when looking at the details of their behaviour, some clusters of formulas emerge with very high levels of agreement in determining the winning bid in these real tenders. Outside of these clusters, there a few formulas that relatively often choose winners no other formula chooses. Using the Tables presented in this paper, practitioners can compare the behavior of the bid evaluation formula(s) they currently use and do not use, and see the difference it makes which formula is chosen.

A buyer who is considering what bid evaluation formula to use to select the economically most advantageous tender, may first want to consider whether or not a formula offers protection against winners with extremely high prices. If a formula rewards quality very well and does not penalize high prices, there are chances that a bidder who stands out in quality can theoretically charge any price it wants. This knowledge may be exploited by a bidder. Our analysis has shown that formula 3 has a strong tendency to maintain the winning bid even if its price is increased 50-fold. This formula should probably not be used, or used with extreme care, unless there are external factors which will prevent bidders from putting in extremely high prices in the first place. This formula offers relatively little protection against winners with extremely high prices because it has concave indifference curve.

When choosing between an absolute and a relative formula, buyers should consider the risk of a mismatch between reference price or price range and market prices against the risk of a ranking paradox. The ranking paradox is avoided when choosing an absolute formula, but of 14 absolute formulas we have studied, all except four need some extra input such as reference price or price range. Choosing one of these absolute formulas creates the burden for the tendering entity of requiring pre-tender market price knowledge and with it, the risk of deviations between expectation and reality.

As for the ranking paradox, we considered a ‘number 1 drop-out ranking paradox’ and a ‘number n drop-out ranking paradox’. In our study of 382 real tenders, we show that the practical occurrence of the ranking paradox for most formulas is small. For most formulas, ranking paradox seems to be more of a theoretical notion rather than a real risk in practice. Formulas 3 and 12 show relatively higher risks of the ranking paradox.

We believe that special attention should be given to formulas with non-linear iso-utility curves. A linear relationship between price and quality implies that incremental units of quality have a constant value. A non-linear relationship implies that units of quality vary in value depending on the level of quality of the individual offer. Concave curves imply that the buyer values consecutive units of quality more and more, while convex curves imply the opposite. We argue that the various criteria defining quality are implied to be fungible if the weighted multi-criteria analysis is used to measure quality. Therefore, we believe it is more clear and fair for buyers and suppliers to use formulas with straight iso-utility curves.

We also considered four scenarios to demonstrate how different the 38 formulas are when it comes to reconciling the weights of price and quality. In this section we also discussed the disadvantages of including the worst price or quality in the formula, as well as the average and median price or quality. This analysis clearly showed that five formulas with such elements of ‘bid spread’ (formulas 2, 3, 13, 19, and 24) are very price sensitive when bid prices do not differ much.

Obviously, the outcome of the tender does not only depend on the choice of a formula. In fact, many practitioners rightfully assume that the outcome depends more so on the choice of weights for price and quality. However, we found that the choice of a formula and the choice of weights for quality and price interact to determine the outcome of the tender.

Some formulas could not be tested because they require information that is particular to the tender. This paper provides suggestions for the kind of simulations a tendering entity can perform to study the behaviour of such formulas when such particular information is known to them. This research should help buyers to challenge the formula(s) they currently use and/or to discover and choose a formula that best serves the goals of their organizations. Lastly, neither the collection of 38 different formulas, nor the dimensions to evaluate and test EMAT formulas presented in this paper are likely to be complete. We invite other researchers and practitioners to add to the pool of formulas as well as to add evaluative tests and experiments to better understand the behaviour of various bid evaluation formulas.

NOTES

1. In VBA (Value Based Awarding), the criteria get a monetary value assigned to them rather than a % weight.
2. In very rare cases, there is only one non-price criterion.

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APPENDIX A

i - individual bid	P_{Avg} - average price of all bids
$W_{Quality}$ - weight of quality	W_{Price} - weight of price
Q_i - quality of each individual bid	P_i - price of each individual bid
Q_{Best} - highest quality of all bids	P_{Best} - lowest price of all bids
Q_{Ref} - reference quality	P_{Ref} - reference price
Q_{Median} - median quality of all bids	P_{Median} - median price of all bids
$Q_{Set\ Max}$ - highest possible quality	$P_{Set\ Max}$ - upper end of predefined price range
b - value per quality point, user-defined parameter	$P_{Set\ Min}$ - lower end of predefined price range
n, s, α, β - user-defined parameters	P_{Max} - highest price of all bids
Note: Some of the formulas are named after an organization, which does not imply that the organization is still using this particular formula. Some other formula names refer to the author(s) who have published the formula in their work. Source for the formula is given in the square brackets.	

Formulas

1. Lowest Bid Scoring* [Dimitri, Piga and Spagnolo (2006)]

$$Score_i = \frac{P_{Best}}{P_i} W_{Price} + Q_i W_{Quality}$$

2a. Highest Bid - Lowest Bid Scoring* [Dimitri, Piga and Spagnolo (2006)]

$$Score_i = \frac{P_{Max} - P_i}{P_{Max} - P_{Best}} W_{Price} + Q_i W_{Quality}$$

2b. Based on Bid Spread [Waara and Brochner (2006)]

$$Score_i = \frac{P_i - P_{Max}}{P_{Max} - P_{Best}} W_{Price} - Q_i W_{Quality}$$

Formula 2b is a variant of 2a leading to exactly the same results in our analyses. We present both versions to make it as easy as possible for practitioners to recognize the formula they may be familiar with.

3. Average Scoring* [Dimitri, Piga and Spagnolo (2006)] $$Score_i = \begin{cases} W_{Price} + Q_i W_{Quality} & if \; P_i < P_{Avg} \\ \frac{P_{Max} - P_i}{P_{Max} - P_{Avg}} W_{Price} + Q_i W_{Quality} & otherwise \end{cases}$$
4. Based on Average Bid [Waara and Brochner (2006)] $$Score_i = \frac{P_i}{P_{Avg}} W_{Price} - Q_i W_{Qualit}$$
5. Maximum Price Deviation Model [Waara and Brochner (2006)] $$Score_i = \left(1 - \frac{P_i}{P_{Max}}\right) W_{Price} + Q_i W_{Quali}$$
6. Utility Index [Negometrix, personal communication] $$U_i = \frac{\left(1 - (Q_{Best} - Q_i)\frac{W_{Quality}}{W_{Price}}\right) P_{Best}}{P_i};$$ $$Score_i = P_i \left(\frac{\max(U_1, \cdots, U_N) - U_i}{\max(U_1, \cdots, U_N)} \right)$$
7. Coventry City Council [Coventry City Council, URL no longer available] $$Score_i = \frac{P_{Best}}{P_i} W_{Price} + \frac{Q_i}{Q_{Best}} W_{Qualit}$$
8. European Organization for Nuclear Research (CERN) [CERN Engineering Department, URL no longer available] $$Score_i = W_{Price} + 0.5\left(1 - \frac{P_i}{P_{Best}}\right) + Q_i W_{Quality}$$
9. Domb & Tsur [Uria Domb and Ofer Tsur, personal communication] $$Score_i = \frac{Q_{Worst} + (Q_i - Q_{Worst})\frac{W_{Quality}}{W_{Price}}}{P_i}$$

10. Mercer [Negometrix, personal communication] $$Score_i = \begin{cases} \left(1 - \frac{P_i - P_{Best}}{P_{Best}}\right) W_{Price} + Q_i W_{Quality} & if \ \frac{P_i - P_{Best}}{P_{Best}} \leq 1 \\ Q_i W_{Quality} & otherwise \end{cases}$$
11. Scottish Government [Scottish Government] $$Score_i = \left(0.5 - \frac{P_i - P_{Avg}}{P_{Avg}}\right) W_{Price} + Q_i W_{Quality}$$
12. Waterschap Brabantse Delta [Negometrix, personal communication] $$Score_i = \left(1 - \frac{P_i - P_{Best}}{P_{Best}}\right) W_{Price} + Q_i W_{Qualit}$$ If the price difference between the lowest bid and the 2nd lowest bid is greater than 20%, then the 2nd lowest bid gets 80% of price points of the lowest bid and the score of consecutive bids is calculated according to the formula below. $$Score_i = \left(1 - \frac{P_i - P_{2nd\ Best}}{P_{2nd\ Best}}\right) W_{Price} + Q_i W_{Quality}$$
13. Score by Rank* [Smith (2010)] $$Score_i = p W_{Price} + Q_i W_{Quality}$$ p is the score on price. The highest price bid earns 0 and the lowest priced bid 1 point on the price score. All other price scores are placed at equal increments between 0 and 1 according to their ranking on price.
14. Chen 2* [Chen (2008)] $$Score_i = \left(1 - 0.5 \frac{P_i}{P_{Best}}\right) W_{Price} + Q_i W_{Qualit}$$
15. Chen 3* [Chen (2008)] $$Score_i = \left(1 - 0.5 \frac{\log\left(\frac{P_i}{P_{Best}}\right)}{\log(2)}\right) W_{Price} + Q_i W_{Quality}$$

16. Chen 4* [Negometrix, personal communication]

$$Score_i = \begin{cases} \left(1 - 0.5\dfrac{\log\left(\dfrac{P_i}{P_{Best}}\right)}{\log(2)}\right) W_{Price} + Q_i W_{Quality} & if\ (.) W_{Price} > 0 \\ Q_i W_{Quality} & otherwise \end{cases}$$

Formula 16 is a general case of formula 15.

17. UfAB II-Formel [BMI (2012)]

$$Score_i = \frac{Q_{Best} P_{Best}}{P_i} W_{Price} + Q_i W_{Quality}$$

18. UfAB Medianmethode [BMI (2012)]

$$Score_i = \frac{Q_i}{Q_{Median}} W_{Quality} - \frac{P_i}{P_{Median}} W_{Price}$$

19. Pauw & Wolvaardt* [Pauw and Wolvaardt (2009)]

$$Score_i = \frac{P_{Max} - P_i}{P_{Max} - P_{Avg}} W_{Price} + Q_i W_{Quality}$$

20. Based on the Average Price* [PSIBouw (2007)]

$$Score_i = \left(1 - \frac{P_i - P_{Best}}{P_{Avg}}\right) W_{Price} + Q_i W_{Quality}$$

21. Based on the Lowest Price* [PSIBouw (2007)]

$$Score_i = \frac{2P_{Best} - P_i}{P_{Best}} W_{Price} + Q_i W_{Quality}$$

22. Quotient Verdeling 1 [Negometrix, personal communication]

$$Score_i = \frac{P_{Avg} - P_i + P_{Best}}{P_{Avg}} W_{Price} + \frac{Q_i}{Q_{Best}} W_{Quality}$$

23. Quotient Verdeling 2 [Negometrix, personal communication]

$$Score_i = \frac{2P_{Best} - P_i}{P_{Best}} W_{Price} + \frac{Q_i}{Q_{Best}} W_{Quality}$$

24. Quotient Verdeling 3 [Negometrix, personal communication]

$$Score_i = \frac{P_{Max} - P_i}{P_{Max} - P_{Best}} W_{Price} + \frac{Q_i}{Q_{Best}} W_{Quality}$$

25. Tennet [Negometrix, personal communication] $Score_i = P_i + P_i\left(1 - \frac{Q_i}{Q_{Set\ Max}}\right)\frac{W_{Quality}}{W_{Price}}$
26. Chen 1** [Chen (2006)] $Score_i = \frac{P_i}{P_{Set\ Max}}W_{Price} + \frac{Q_{Set\ Min}}{Q_i}W_{Quality}$
27. Kuiper's Superformula** [Hans Kuiper, personal communication] $Score_i = \sqrt[n]{\left(\frac{P_i}{P_{Q=1}}\right)^n + \left(\frac{1-Q_i}{1-Q_{P=0}}\right)^n}$ $P_{Q=1}$ is a pre-defined reference price for the highest imaginable quality. $Q_{P=0}$ is a pre-defined reference quality for the lowest imaginable price.
28. PSIBouw Value Based** [Negometrix, personal communication] $Score_i = P_i - bQ_i$
29. ISZF [Negometrix, personal communication] $Score_i = \frac{P_i^{W_{Price}}}{Q_i^{W_{Quality}}}$
30. Belastingdienst S-curve*,** [Negometrix, personal communication] $Score_i = \left(1 - \left(\frac{1}{1+\exp(100\alpha(\beta - P_i))}\right)\right)W_{Price} + Q_i W_{Quality}$
31. Kuiper 1** [Kuiper (2009)] $Score_i = P_i - \frac{W_{Quality}}{W_{Price}}P_{Ref}\left(1 - \frac{Q_{Ref}}{Q_i}\right)$
32. Kuiper 2 [Kuiper (2009)] $Score_i = \frac{P_i}{Q_i}$
33. Kuiper 3** [Kuiper (2009)] $Score_i = \left(2 - \frac{P_i}{P_{Ref}}\right)W_{Price} + \left(\frac{Q_i}{Q_{Ref}}\right)W_{Quality}$

34. Telgen** [Negometrix, personal communication] $Score_i = \frac{P_{Set\ Max} - P_i}{P_{Set\ Max} - P_{Set\ Min}} W_{Price} + Q_i W_{Quality}$
35. Argitek [Negometrix, personal communication] $Score_i = \frac{P_i}{Q_i^{\frac{W_{Quality}}{W_{Price}}}}$
36. Bergman and Lundberg 1** [Bergman and Lundberg (2013)] $Score_i = P_i + b\left(Q_{Set\ Max} - Q_i\right)$
37. Bergman and Lundberg 2** [Bergman and Lundberg (2013)] $Score_i = P_i\left(1 - bQ_i\right)$
38. Bergman and Lundberg 3** [Bergman and Lundberg (2013)] $Score_i = P_i\left(1 + b\left(Q_i - Q_{Set\ Max}\right)\right)$

Notes: *=Formula adjusted by the authors to allow for the weights of price and quality and quality score.
**= Formula uses a user-defined parameter that is set by the buyer and known to the bidder.