# REPUTATIONAL COMMITMENTS AND COLLUSION IN PROCUREMENT

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**ABSTRACT**. When gains from trade exist both along contractible and non-contractible dimensions such as with R&D for innovative projects and procurement is repeated, non-contractible gains from trade can be realized through reputation/relational contracting because the buyer can threaten replacement of sellers that perform poorly along non-contractible dimensions. In such a dynamic procurement framework, keeping the optimal number of eligible sellers and contract duration endogenous, we find that: a) there is a general trade-off between reputation for quality and collusion because shorter contracts - more frequent re-auctioning - together with reduced competition - small number of eligible suppliers - facilitate non-contractible quality provision, but also collusive agreements among suppliers; and b) when non-contractible quality and variability in suppliers' efficiency are both important, short contract duration and a collusive agreement between a few eligible sellers may be nevertheless desirable for the buyer and also welfare maximizing. Indeed, collusion or cooperation among firms (e.g. firms consortia) may boost non-contractible dimensions.

## INTRODUCTION

Non-contractible dimensions are present in different measure in every economic exchange.[1] For example in procurement of complex IT services it is often impossible to fully specify all the requirements that are of value for a buyer. R&D activities are often at the frontier of technological knowledge, effort towards improvements is difficult to

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measure and it is often impossible, to check whether the contractor has behaved as promised. Similarly, in health procurement it is often the case that the more critical is quality of procured goods and services, the more difficult is to correctly specify required properties of services to be procured and whether procurement comply with them.

It is well known that when these non-contractible dimensions are important in terms of gains from trade, letting suppliers compete on price - say, in an auction - may lead to a very inefficient outcome, for a buyer and in general.[2] Exchanges, however, are often regularly repeated, particularly in procurement where buyers need to be served over time. Reputational forces may then help governing transactions on non-contractible dimensions. An opportunistic supplier that overstates the non-contractible quality of an experience good or that purposely reduces non-verifiable but ex-post observable qualitative aspects to cut costs and bust profits can be punished by its buyer(s). Clearly, this cannot take place under any infringement of contractual terms, but the buyer can exercise some of his discretion to deliberately hinder an "unfaithful" contractor. With this respect, the most effective and, probably, the most natural punishment consists in excluding the supplier from (some) future trade(s). This form of punishment is certainly available in private contracting where the buyer his generally free to exclude any buyer from its selecting process. In the case of public procurement this decision is still viable to some extent, but it may be partially limited by ruling laws which often restrict civil servants' discretion so as to avoid corruption. However, some national legislations do provide room for discretion in exclusion of dubious providers and leave some accountability in the hands of public buyers.[3]

This paper analyzes repeated procurement processes (recurrent auctions or other forms of search) where non-contractible dimensions are an important source of gains from trade. In particular, quality is non-contractible in our analysis and will be interpreted in broad sense capturing all value-enhancing decisions that a supplier is free to take during the contract execution and which the buyer observes but cannot directly enforce because they are not verifiable. Among the most interesting instances of this type of decisions we list many type of investments for innovation, R&d activities and, in general, effort. Certainly, there are normally other dimensions in the relationships that are under the control of the supplier but that are contractible and can then be governed with a proper design of the contract by the buyer and with an adequate price paid for procurement. In addition, the buyer can also condition the procurement environment. In particular, she can decide to vary the amount of search done before proposing a contract, namely it has the possibility to fix the number of suppliers in the pool admitted to participate at the recurrent auctions. Furthermore, she can also decide the length of the procurement contract and consequently also the frequency of the selection procedure. As previously discussed, we will also allow the buyer to punish with exclusion from future procurement contests firms that have decided to offer low levels of quality along the contractual relationship.

We show that, in a dynamic procurement process the buyer may want to restrict the number of potential trading partners at the cost of reduced screening and more expensive procurement. This is done to boost non-contractible quality. Indeed, by restricting competition, the buyer leaves firms sufficient future rents so that they can find profitable to build reputational commitments for future interactions and prefer to provide acceptable levels of non-contractible quality in the current relationship (and refrain from moral hazard). Duration of supply contracts is also a crucial aspect in dynamic procurement. Abstracting from (important) technological aspects such as the rate of obsolescence, a shorter duration of supply contracts implies more frequent re-selection or search. We then show that with higher frequency of interaction it is easier for a buyer to obtain high non-contractible quality levels from sellers by threatening exclusion from future trade. Indeed, with more frequent contracting the threat of exclusion is closer in time and gains from "cheating" are smaller so that larger implementable quality can be expected. Furthermore, shortening contract length also reduces the buyer's risk to be locked-in with undesirably low quality but at the same time these desirable effects for the buyer have to be contrasted with the buyer's costs of running and organizing more frequent auctions and with firms' inability to spread possible fixed (and relation-specific) costs of procurement over a longer contract.

Considering pros and cons we then show that in repeated procurement a buyer may profit from suppliers' stake on future profits and induce larger non-contractible quality by restricting the pool of firms admitted to compete for the procurement contract and reducing the contract length or, which is equivalent, increasing the frequency of recurrent auctions. With respect, we also show that these two important dimensions of procurement which would be normally independent instruments for the buyer in case of fully contractible quality, turn out to be closely related (either in terms of complementarity or substitutability) when quality is non contractible.

Non-contractible dimensions thus induce the buyer to run frequent auctions within a limited number of potential suppliers. However, it is also well known that this environment that the buyer designs to (at least partially) govern non-contractible quality is also the most favorable environment for inducing and sustaining collusive behavior between the selected suppliers. Hence, we illustrate a rather general and possibly disappointing trade-off between reputation for non-contractible quality and collusion in dynamic procurement. Longer duration of supply contracts - less frequent auctions - together with larger pool of competing suppliers both deter collusion among eligible suppliers but also reduce non-contractible quality levels obtainable from them. Symmetrically, shorter contracts - more frequent auctions - and a smaller pool of suppliers both facilitate suppliers' collusion but also the enforcement of non-contractible quality standards. [4]

This trade-off may show up being rather disappointing because it seems to mark limits on the remedies that can be put at work for non-contractible quality provision. However, our analysis clarifies that collusion itself can directly interact with firms' incentives to provide non-contractible quality. In fact, by increasing the selling price, collusion clearly increases the expected gains from participating in procurement auctions which, as usual, can also be seen as the cost of being excluded when the buyer reacts to low levels of non-contractible quality. Hence, we show that the seeming trade-off between non-contractible quality and collusion, that naturally seems to emerge in repeated procurement, may in fact reveal to be only apparent because larger future rents associated with collusion make firms ready to offer larger non-contractible quality. This somehow provocative result suggests that the buyer may not be necessarily concerned by the spontaneous formation of cartels and may even foster establishment of (legal) consortia among firms that may alternatively compete. Indeed, reinterpreting our result on collusion and non-contractible quality, cooperation among firms in the form of consortia or joint-ventures are made stable by frequent interaction among a limited number of potential competitors and, in addition, they may also be expected to provide larger levels of quality. Hence, lessening competition by reducing the pool of potential suppliers and shortening contract duration that both induce a collusive or legal agreement between eligible suppliers may leave the buyer better off. For example, this suggests that in the procurement of innovation cooperation among firms and in the limit also collusion, do not necessarily hinder innovation when R&D activity is not fully-contractible.

Further exploring this point, we also note that it is often the case in repeated procurement that participating firms have better knowledge on their relative abilities and efficiency than the buyer. Hence, in addition to the previous effects of agreements among firms, consortia and collusion may also have a desirable sorting effect whereby more efficient firms may be selected for supplying. This may not necessarily translate into lower prices for the buyer, and certainly does not in the case of collusion. Rather, larger efficiency in production in these cases tend to increase firms rent and then further boosts implementable non-contractible quality.

We then conclude our analysis discussing how a buyer concerned by non-contractible quality, cost of procurement and, possibly, also directly by efficiency in supplying should design the procurement environment (i.e. the number of competing firms and the exclusion rule for low quality provision) together with the contractible dimensions of the relationship (i.e. the contract duration and auction frequency) to effectively compose its objectives.

**Relation with the literature**

Our work is related to several strands of literature. Manelli and Vincent (1995) analyze the adverse effects of competition in procurement with non-contractible quality. They cast their model in terms of adverse selection (sellers are of different quality which is reflected in the produced good), focus on single transaction, and show that when gains from trade are concentrated on non-contractible dimensions, auctions deliver the worst possible outcome, as they select firms producing the good at the lowest cost but with the lowest level of non-contractible quality. Sequential take-it-or-leave-it offers to randomly selected sellers is then a better mechanism. We clearly differentiate from this paper by considering an environment where quality is an effort decision of procuring firms and not an innate characteristic of suppliers and also considering a dynamic context with repeated procurement. Reaching similar conclusions on the desirability of limited competition but for a different reason than the "bad selection" effect in Manelli and Vincent (1995), our paper complements their analysis.[5] In a different context where unexpected contingencies may make renegotiation necessary, Bajari and Tadelis (2001) also show that with relevant non-contractible dimensions auctioning with fixed-price contracts may be dominated by bilateral negotiation over cost-plus contracts. The literature on optimal procurement of innovative goods and services, where pre-auction non-contractible R&D investments are crucial, including for example Taylor (1995), Fullerton and McAfee (1999), and Che and Gale (2003), is also related to our work. In these analyses even though procurement is not repeated, limiting participation is optimal, as in our model, but for a different reason than reputational effects. Reducing the number of participant increases each participant's probability of winning the award, and thereby encourages pre-auction non-contractible R&D effort-investment.

Our environment on repeated purchase with non-contractible quality relates our analysis to the industrial organization literature on reputation and competition which studies whether firms' reputational commitments to high non-contractible quality can be compatible with a competitive environment and that has been initiated by the seminal works on experience good markets of Macaulay (1963), Klein and Leffler (1981), Shapiro (1983), and Allen (1984). These early analyses were concerned with the compatibility of "quality-assuring" reputational equilibria - requiring rents that make the effort of maintaining reputation worthwhile also with free entry in the market - but did not analyze in detail firms' competitive interaction (firms' incentives to steal business from each other).[6] Stiglitz (1989) also raised the question how could reputation be compatible with perfect competition that should eliminate any future supracompetitive gains. More recently, Kranton (2003) offers a model that captures this dilemma. In presence of moral hazard on quality and competition, high quality equilibria are unfeasible and he suggests restricting competition in industries where non-contractible quality is important. In a different model Bar-Isaac (2005) confirms Kranton and Stiglitz's view at the limit, but shows that at intermediate levels of competition a further increase in competition (number of firms and substitutability) may well increase equilibrium product quality. Hoerner (2002) offers the first elegant answer to Stiglitz's question: in his model with heterogeneous consumers, adverse selection and moral hazard, high prices signal high quality and make competition compatible with (in fact necessary for) reputation to work.[7]

In the context of procurement Laffont and Tirole (1993, chapter 4) explain in a two-stage model that when demand is fixed or inelastic, as in the case of many procurement situation in which the buyer buys a fixed amount, one way to stimulate quality provision is linking future business to current quality. Reputation can be exploited to threaten suppliers, especially when the buyer can potentially deal with many suppliers. Kim (1998) and Doni (2004) also study a repeated procurement auctions with moral hazard on non-contractible quality, and show that it may be good to restrict participation and threaten exclusion if the level of non-contractible quality is too low. Even if Shapiro (1983) noted that the frequency of interaction may facilitate the operation of reputational mechanisms with experience goods, the effects of contract duration and auction frequency have been largely neglected in the procurement literature. Contrary to these papers, we consider the possibility to adjust the frequency of procurement, its interaction with the decision to restrict the pool of potential suppliers and also the potential of cartels and consortia in sorting the several trade-offs between quality, procurement costs and efficiency.[8] Ellman (2006) analyses a contract length trade-off associated to the presence of investment which becomes specific to the contractual relationship but does not look at the role of threats in case of repeated purchase.[9]

Our paper also contributes to the literature on incomplete explicit contracts (the "Hart and Moore paradigm"), and in particular on how dynamic interaction allows to complement these by implicit/relational contracts, from the early contributions of Bull (1987) and McLeod and Malcomson (1989, 1998), to the recent ones like Levin (2003) and Fuchs (2005). As in our paper, the focus of this literature is how relational contracting allows parties to enforce and govern agreements on observable but not verifiable dimensions (effort, investment, quality). Within this rich and growing literature, our work is closest in spirit to the contributions more directly focussing on the interaction between explicit and implicit contracts, i.e. on how explicit contracts should be structured or modified to optimize the joint outcome of explicitly contracted dimensions and implicit effects, like Baker, Gibbons and Murphy (1994, 2002), Pearce and Stacchetti (1998), Halonen (2002), Che and Yoo (2001), Blonski and Spagnolo (2003), and Rayo (2004), among others. To our knowledge, none of these studies considers how the design of procurement process may be influenced by relational contracting, nor deals with how repeated screening through auctions, the number of eligible suppliers, the length of explicit contracts, and firms cooperation interact with relational quality commitments.[10]

We conclude this literature review with noticing that there are at least two approaches to reputation in markets, as convincingly emphasized by Bar-Isaac (2003). A first approach views "reputation as beliefs" where uninformed players infer intrinsic qualities of contracting parties by their behavior so that a firm's reputation consists in buyer's beliefs about its quality-type. See for example Kreps and Wilson (1982), Milgrom and Roberts (1982). A second non-exclusive view, which is the one employed in our paper and is related to the previously discussed literature on markets with experience goods and implicit contracting, considers reputation as a self-sustaining commitment to provide desirable but non-contractible quality by a credible threat on the side of the other partner in case reputational commitment is violated.

The rest of the paper is organized as follows. Section 2 presents the model setup. Section 3 analyzes procurement and quality with competing firms. Section 4 discusses the effect of collusion on implementable quality. Section 5 illustrates optimal procurement. Section 6 extends the base model and discusses its main assumptions. Finally, Section 7 concludes.

## MODEL SETUP

A buyer needs to procure a unit of a good (or a service) at any period and she cares for the quality of supply. The per-period valuation of the good $V(q)$ is increasing in the procured quality $q \geq 0$, with $V(0) = 0$.[11] Amongst the $N$ potential suppliers, any firm $i$ can procure a unit of the good with quality $q_i$ by incurring in a per-period cost $\theta_i + \psi(q_i)$ where $\theta_i \in \Theta \equiv [\underline{\theta}, \overline{\theta}]$ is an (in-)efficiency parameter and $\psi(\cdot)$ is a positive real valued function increasing in $q_i$, with $\psi(0) = 0$. The per-period (social) value of quality $V(q) - \psi(q)$ is concave in $q$, time horizon is infinite and all the players have a constant common discount factor equal to $\delta \leq 1$.

A procurement contract determined by the buyer specifies the number of procurement periods $x \geq 1$ and the per-period payment $p$ that the supplier receives for any of the $x$ periods contemplated by the contract. Supplied quality in any contractual relationship is publicly observable but not verifiable and then it is not contractible. Contracts that last more than one period cannot be unilaterally renegotiated (i.e. reneging is ruled out by law), but bilateral renegotiation is admissible.

The buyer is not fully informed on each firm's cost $\theta_i$ so that she may run an auction for any awarding process that selects a supplier. An auction awards a procurement contract to the bidder that offers the lowest acceptable bid $b_i$ for the contract and maps the vector of all bids $\mathbf{b}$ into the payment $b_w$ for contract execution. In the rules of the auction, the buyer may also set a reservation price $r \geq 0$ so that acceptable bids must satisfy $r \geq b_i$ (if $r < b_i$ for any $i$, the buyer does not award the contract) and she may also decide to limit to $n \leq N$ the number of bidders that are eligible and admitted to the auction process.[12] In the following, we will continue to describe the awarding process as an auction also in the extreme case where $n = 1$ so the relationship is in fact bilateral.

The firm that is awarded the contract sets the level of quality it will provide once and for all the duration $x$ of the contract.[13] Quality is not contractible so that the buyer cannot claim contract infringement on the basis of inadequate quality. Nevertheless, the buyer may react to a low procured quality with decisions that affect future contractual relationships with the supplier. In particular, if the quality provided by the auction winning firm does not satisfy the buyer, in a sense that we now clarify, then the latter can discretionally exclude the contractor for some future auction rounds.[14] We model this exclusion rule with a *minimum quality requirement* $\underline{q}$ so that if the firm procures a good of quality $q < \underline{q}$, the buyer can discretionally decide to exclude this seller for the next $T \geq 0$ auctions.

The relationship between the buyer and any potential contractor is thus composed by some terms that are verifiable (and court-enforceable), namely contract duration $x$ and remuneration $b_w$, the reservation price $r$ and the number of eligible bidders $N$, together with terms that are non-contractible and discretionary, the procured quality $q$ and the exclusion rule $\sigma \equiv (\underline{q}, T)$.

Finally, running any procurement contract implies a contract-specific investment with a total (for the buyer and the supplier) costs $K \geq 0.$ For example, $K$ may be a fixed set-up cost that each contractor incurs to procure the good of whatever quality and that has to be paid anew in any contractual relationship. Alternatively, it may also be the buyer's cost for organizing any auction or bargaining with a new supplier. For the sake of concreteness we will often use the latter interpretation.

In this set-up, a strategy for the buyer thus consists in, once and for all, (i) setting the awarding rules and the contract length, which we will indicate as the contractible elements of the procurement relationship $(n, r, b_w, x)$, and (ii) publicly announcing the exclusion rule $\sigma$ which may help the buyer governing non-contractible behavior of suppliers.[15] A strategy for any firm $i$ is composed by a participation decision and a bid for any awarding process and a decision on procured quality for any contract it is awarded.

The **timing of the game** is as follows.

$t = -2$: The buyer sets the contract length $x$, the number of eligible bidders $n$, the reservation price $r$ and announces the exclusion rule $\sigma = (\underline{q}, T)$.

$t = -1$: The buyer randomly selects the $n$ firms amongst the $N$ potential sellers.

$t = 0$: An infinite repetition of the following stage game (or auction game) takes place.

**Stage Game**

*At time* $t_1 \geq 0,$ *an auction publicly awards the procurement contract, the winning contractor* $i$ *obtains the payment* $b_w$ *and sets procured quality* $q_i$*;*

*At any period* $t \in [t_1, t_1 + x - 1]$ *the winner procures the good of quality* $q_i$;

*At time* $t_1 + x,$ *if* $q_i < \underline{q}$ *the supplier* $i$ *is excluded for next* $T$ *awarding processes and possibly replaced by a new firm,* *otherwise the pool $n$ remains unchanged and a new stage game starts.*

Although relatively simple, the dynamic described game may turn to be intractable in its most general form. We will temporarily limit our attention to a simplified version based on some (strong) working assumptions. Then, having emphasized the main ideas and results in this clear-cut environment, in Section 6 we will discuss these assumptions and alternative model specifications.

- **Assumption 1** *(i) In each period, the cost parameter $\theta_i$ of any firm $i$ is drawn anew from a time-invariant and independent distribution $f(\theta_i)$ ($>0$ for any $\theta_i \in \Theta$) which is common knowledge; (ii) Firms are fully informed.*
- **Assumption 2** *The number of potential suppliers $N$ is infinite.*
- **Assumption 3** *The buyer needs to be served at any period, otherwise she obtains a per-period payoff equal to $-k$ with $k >> 0$.*

Assumption 1 (i) could be substituted by considering a $\theta_i$ which is drawn anew at any auction stage but that remains the same for the contract duration $x$. This would not qualitatively alter our results. On the contrary, a model where each firm is characterized by a permanent level of efficiency $\theta_i$ forever would be much less tractable because the buyer would then learn firms' efficiency auction after auction and firms would anticipate that their actions signal information. Also assumption 1 (ii) is certainly not without loss of generality. It greatly simplifies our analysis, but as we will further discuss in Section 6, our results and the underlying trade-offs seem to be robust to asymmetrically informed firms. Assumption 2 will be relaxed and discussed in Section 6 as well as assumption 3 which simplifies the analysis of the reservation price because interrupting the flow of goods is extremely costly for the buyer.

Note also that to simplify the exposition we have not explicitly modelled the many other elements of the procurement relationship that are contractible. However, a more general interpretation of our model is that the buyer evaluates all these contractible elements with a scoring rule that is here simply represented by firms' bids, scores for all contractible elements. In Section 6 we will also explore the possibility for the buyer to use scoring rules on non-contractible quality, which in the current model setup are irrelevant.

## IMPLEMENTABLE QUALITY WITH COMPETING SUPPLIERS

Consider a buyer' strategy comprising a pool $n$ of eligible firms, a contract length $x$, and a given exclusion rule $\sigma$. For a given awarded contract in which the supplier offers a quality $q$ and receives the payment $b_w$, the buyer' surplus is

$$\frac{1-\delta^x}{1-\delta}V(q)-b_w-K.$$

Hence, in case at any auction any winning firm receives the same $b_w$ and offers the same quality $q$, the buyer's total surplus is

$$S(q,x,n)=\sum_{t=0}^{\infty}\delta^{tx}\left[\frac{1-\delta^x}{1-\delta}V(q)-b_w-K\right]=\frac{1}{1-\delta}\left[V(q)-b_w\right]-\frac{1}{1-\delta^x}K$$

where the dependence on $n$ implicitly takes place through the price $b_w$, as it will be clear in the sequel.

For any vector of $n$ firms' cost-efficiency $\theta\equiv(\theta_1,...,\theta_n)$ in a given period, let $\theta'(n)\equiv\min\{\theta_1,...,\theta_n\}$ be the cost parameter of the most efficient firm in the pool of $n$ eligible suppliers (i.e. the first order statistics of $\theta$), with $\theta'(n)$ clearly decreasing in $n$. Then, maximal surplus that can be generated by a contract of length $x$ and procured quality $q$ is

$$\frac{1-\delta^x}{1-\delta}\left[V(q)-E[\theta'(n)]-\psi(q)\right]-K$$

where $E[\cdot]$ is the expectation operator over the realization of $\theta$ and, assuming a repetition of the same contract with the same quality, overall welfare is

$$W\left(q,x,n\right)=\sum_{t=0}^{\infty}\delta^{tx}\left\{\frac{1-\delta^{x}}{1-\delta}\left[V(q)-E\left[\theta'\left(n\right)\right]-\psi\left(q\right)\right]-K\right\}=\frac{1}{1-\delta}\left[V(q)-E\left[\theta'\left(n\right)\right]-\psi\left(q\right)\right]-\frac{1}{1-\delta^{x}}K.$$

Our simple framework is stationary for the buyer in the sense that, except for the supplier’s decision to comply or not with quality, firms’ efficiency is not persistent and learning on efficiency does not occur. Hence, the buyer’s optimal strategy is time invariant and she sets the contractible elements of the procurement process and the rule $\sigma$ with quality requirement once and for all stages of the game.

Consider now the exclusion rule $\sigma$ set by the buyer. We will consider the toughest exclusion rule available to the buyer that punishes disloyal and cheating firms that provide quality $q<\underline{q}$ with exclusion forever from future auctions. This exclusion rule is credible in the current framework because the (infinite) firms are ex-ante identical, so that replacing a firm comes at no cost for the buyer and it clearly constitutes an optimal penal code in the sense of Abreu (1988).[16]

Being quality not-contractible, any firm $i$ that wins an auction may choose to satisfy the quality requirement $\underline{q}$, i.e. to provide a quality $q\geq\underline{q}$ or not. Clearly, none of the firms has incentive to provide a quality larger than the minimum requirement because it will not be credited for extra quality so that in equilibrium quality will be either $q=\underline{q}$ when the firm’s decision is to satisfy the quality requirement, or $q=0$ if the suppliers decides to cheat on quality. If the winning firm decides to comply with the buyer’s quality requirement, it obtains an expected profit equal to

$$b_{w}-\theta-C(q)+E\left[\Pi\mid q\geq\underline{q}\right]\frac{\delta^{x}}{1-\delta^{x}}$$

where

$$C(q)\equiv\delta E[\theta]\frac{1-\delta^{x-1}}{1-\delta}+\psi\left(q\right)\frac{1-\delta^{x}}{1-\delta}$$

is the sum of expected cost of production for the $x-1$ periods of supply after the first one (recall that firm $i$ knows only the current realization of

$\theta_i$) and the cost of procuring quality $q$ along the contract, whilst the term $E\left[\Pi \mid q \geq \underline{q}\right]$ represents the expected profits from any future auction of a firm always complying with the quality requirement.[17] Alternatively, then the winning firm may decide to shirk on quality, thus providing $q = 0.$ By so doing the firm saves the quality production costs $\left(1-\delta^x\right)/\left(1-\delta\right)\psi\left(\underline{q}\right)$ but it incurs in exclusion from future auctions, with an overall profit equal to

$$b_w - \left(\theta + \delta \frac{1-\delta^{x-1}}{1-\delta} E[\theta]\right).$$

It is then clear that at any auction, any winning firm will be ready to satisfy the quality requirement if expected future profits are larger than the immediate cost saving it can obtain in the current contractual relationship, i.e.

$$E\left[\Pi \mid q \geq \underline{q}\right]\frac{\delta^x}{1-\delta^x} \geq \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta}. \qquad (1)$$

Hence, in the following we will say that a *minimum quality requirement* $\underline{q}$ *is implementable* if it satisfies condition (1).This reasoning can be further exploited noticing that, by standard arguments on price competition, at any auction taking place at stage $t$ the winning firm will be the most efficient firm, i.e. the one with cost $\theta'\left(n\right)$ for the vector $\theta$ occurring in $t,$ who obtains a contractual price which is equal to the minimum between the reservation price $r$ and the second most efficient firm's cost. Furthermore, the risk of being left with no provision of the required good forces the buyer to set a high reservation price that guarantees procurement for any realization of $\theta$, i.e. $r = \overline{\theta} + C(q)$. Hence, the awarded price $b_w$ can be written as

$$b_w = \theta''\left(n\right) + C(q)$$

where $\theta''\left(n\right) = \min\left\{\theta/\theta'\left(n\right)\right\}$ if the buyer admits at least two firms at the auction stage (i.e. $n > 1$) and $\theta''\left(n\right) = \overline{\theta}$ if she prefers contracting with a single firm (who can ask the reservation price). We then obtain the following Lemma.

**Lemma 1.** *Maximal implementable quality* There exists a maximal implementable quality $\overline{q}(x,n)$ such that a quality requirement $\underline{q}$ is implementable if $\overline{q}(x,n) \geq \underline{q},$ with $\overline{q}(x,n)$ decreasing in $n$ and $x$, nil if either $n=\infty$ or $x=\infty$ and, if $\psi$ is concave, then the negative effect of $x$ ($n$) on $\overline{q}$ reduces with $n$ ($x$).

**Proof of Lemma 1.** As stated in the text, the winning firm prefers to provide the required quality if

$$E\left[\Pi \mid q \geq \underline{q}\right]\frac{\delta^x}{1-\delta^x} \geq \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta}.$$

Now, the probability that a firm $i$ with cost $\theta_i$ is the most efficient firm will be denoted as $\Pr(\theta_i \leq \theta_j, \forall j \neq i \mid \theta_i)$ and, from independence, we have $\Pr(\theta_i \leq \theta_j, \forall j \neq i \mid \theta_i) = \left[1-F(\theta_i)\right]^{n-1}$. Hence, the ex-ante probability of being the lowest cost firm at any stage game is $\Pr(\theta_i \leq \theta_j, \forall j \neq i) = \int\left[1-F(\theta_i)\right]^{n-1} f\left(\theta_i\right) d\theta_i = 1/n.$

Consider now a (possible) equilibrium where all firms are ready to supply the required minimum quality $\underline{q}$. The most efficient firm the wins and, applying the optimal reservation price, in any future auction with cost $\theta$ obtains a rent $\Delta\theta(n) = \theta''(n) - \theta'(n),$ so that

$$E\left[\Pi \mid q \geq \underline{q}\right] = \frac{E\left[\Delta\theta(n)\right]}{n}.$$

Let (implicitly) define $\hat{q}(x,n)$ by

$$E\left[\Delta\theta(n)\right]\frac{\delta^x}{n(1-\delta^x)} = \psi\left(\hat{q}\right)\frac{1-\delta^x}{1-\delta}, \tag{2}$$

It is thus immediate that for any quality requirement $\underline{q} > \hat{q}(x,n)$, any supplier would prefer to shirk on quality so that a quality requirement is implementable only if $\underline{q} \leq \hat{q}(x,n)$. We now also need to show that by offering a price $b_w = \theta''(n) + C(q)$, the most efficient firm indeed wins the auction. In principle a less efficient firm may be able to undercut by offering a lower price and planning to offer a lower quality (alternatively it would be never able to profitably undercut the offer $b_w$). For this it suffices to consider the second most efficient firm who will not undercut the most efficient one if

$$\theta''(n) + C(q) - \theta''(n) - \delta\frac{1-\delta^{x-1}}{1-\delta}E[\theta] - \frac{1-\delta^x}{1-\delta}\psi(0) \leq E\left[\Pi \mid q \geq \underline{q}\right]\frac{\delta^x}{1-\delta^x}.$$

This is clearly equivalent to

$$E\left[\Pi \mid q \geq \underline{q}\right]\frac{\delta^x}{1-\delta^x} \geq \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta}$$

which is the same condition analyzed for the quality provision by the most efficient firm so that undercutting is not profitable if $\underline{q} \leq \hat{q}(x,n)$

Now note that, even if the buyer sets $\underline{q} = \hat{q}(x,n)$, an equilibrium may prevail where all firms supply nil quality. Indeed, suppose all $n-1$ firms plan to offer $q = 0$, then the most efficient firm can win by asking a price $b_w = \theta''(n) + C(0)$ (recall the second most efficient firm here procures $q = 0$) but still providing the required quality $\underline{q}$. In this case, the future expected profits if it wins and abides the quality requirement

$$E\left[\Pi \mid q \geq \underline{q}\right] = \frac{1}{n}\left[E\left[\Delta\theta(n)\right] - \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta}\right]$$

so that it will prefer to provide the required quality if

$$E\left[\Delta\theta(n)\right]\frac{1}{n}\frac{\delta^x}{1-\delta^x} \geq \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta}\left[1+\frac{\delta^x}{n\left(1-\delta^x\right)}\right]. \tag{3}$$

Now let define $\tilde{q}(x,n)$ the $\underline{q}$ that satisfies the condition (3) written as an equality. On the other hand, it is also clear that this behavior by firms can be consistent with an equilibrium only if the winning firm asks a price $b_w = \theta''(n) + C(0)$ which is associated with nil quality because for any higher price the buyer, anticipating a nil quality, would prefer to set $\underline{q} = 0$.

It is immediate that $\hat{q}(x,n) \geq \tilde{q}(x,n)$. Furthermore, for a given minimum quality requirement $\underline{q}$, we have that the level of quality $q$ effectively procured in equilibrium is

$$q = \begin{cases} 0 & \text{if } \underline{q} > \hat{q}(x,n), \\ \{0, \underline{q}\} & \text{if } \hat{q}(x,n) \geq \underline{q} > \tilde{q}(x,n), \\ \underline{q} & \text{if } \tilde{q}(x,n) \geq \underline{q}. \end{cases}$$

which shows that if $\underline{q} > \hat{q}(x,n)$ in any event all firm will prefer to cheat on quality, if the buyer asks for a lower quality $\hat{q}(x,n) \geq \underline{q} > \tilde{q}(x,n)$ then two types of equilibria may prevail so that the buyer ends up either with nil procured quality or with quality $\underline{q}$. Finally, setting a sufficiently low (but positive) quality requirement $\tilde{q}(x,n) \geq \underline{q}$, firms systematically prefer to abide the requirement and the buyer does not risk to end up with nil quality.

Consider now the properties of $\hat{q}(x,n)$ and $\tilde{q}(x,n)$. That $\hat{q}(x,n) = 0$ for $x = \infty$ and any $n$ is immediate from (2). Treating $n$ and $x$ as continuous variables for simplicity, from implicit differentiation we obtain

$$\frac{\partial \hat{q}}{\partial x} = \frac{\left(1+\delta^x\right) Log[\delta]}{1-\delta^x} \frac{\psi\left(\hat{q}\right)}{\psi_q\left(\hat{q}\right)} \leq 0.$$

Also note that $F_{\theta'(n)}(x) = 1-(1-F(x))^n$ with $\frac{\partial F_{\theta'(n)}(x)}{\partial n} \geq 0$ and $F_{\theta''(n)}(x) = 1-(1-F(x))^{n-1}(1+F(x)(n-1))$ with $\frac{\partial F_{\theta''(n)}(x)}{\partial n} \geq 0,$ thus both for $\theta''\left(n\right)$ and $\theta'\left(n\right),$ increasing $n$ amounts to a first order stochastic dominance effect. We then also have $\frac{\partial E[\theta''(n)-\theta'(n)]}{\partial n} \leq 0$ because the larger is $n$ the smaller is the expected difference in term of efficiency between the most and the second most efficient firms and also $\lim_{n\to\infty} E[\theta''\left(n\right)-\theta'\left(n\right)] = 0.$ Hence, the limit for $n \to \infty$ of the left hand side in (2) is zero so that $\hat{q}\left(x,n\right) = 0$ with $n = \infty$ for any $x$ and also

$$\frac{\partial \hat{q}}{\partial n} = \frac{\delta^x\left(1-\delta\right)}{(1-\delta^x)^2}\left[\frac{\partial E\left[\Delta\theta\left(n\right)\right]}{\partial n}\frac{1}{n} - \frac{E\left[\Delta\theta\left(n\right)\right]}{n^2}\right]\frac{1}{\psi_q\left(\hat{q}\right)} \leq 0.$$

Finally, differentiating $\frac{\partial \hat{q}}{\partial x}$ with respect to $n,$ we obtain

$$\frac{\partial^2 \hat{q}\left(x,n\right)}{\partial x \partial n} = \frac{\left(1+\delta^x\right) Log[\delta]}{1-\delta^x} \frac{\psi_q\left(\hat{q}\right)-\psi_{qq}\left(\hat{q}\right)}{\psi_q\left(\hat{q}\right)^2} \frac{\partial \hat{q}}{\partial n}$$

where the second term in the right hand side is negative when $\psi_{qq} \leq 0$ so that $\frac{\partial^2 \hat{q}(x,n)}{\partial x \partial n} \geq 0$ and a larger $n$ ($x$) makes $\frac{\partial \hat{q}}{\partial x}$ ($\frac{\partial \hat{q}}{\partial n}$) less negative and a larger $x$ makes $\frac{\partial \hat{q}}{\partial n}$ less negative.[18] Similarly, for $\tilde{q}\left(x,n\right)$ we have the same properties when $x = \infty$, $n = \infty$ and also

$$\frac{\partial \tilde{q}}{\partial x}=\frac{\left[n+\delta^{2x}(n-1)\right]Log[\delta]}{(1-\delta)\delta^{x}}\frac{\psi(\tilde{q})}{\psi_q(\tilde{q})}\leq 0$$

$$\frac{\partial \tilde{q}}{\partial n}=-\frac{(1-\delta)\delta^{x}}{\psi_q(\tilde{q})}\frac{\left(1-\delta^{x}\right)E\left[\Delta\theta(n)\right]+\left[\delta^{x}(n-1)-n\right]\frac{\partial E\left[\Delta\theta(n)\right]}{\partial n}}{\left(1-\delta^{x}\right)\left[n-\delta^{x}(n-1)\right]^{2}}\leq 0$$

with $\frac{\partial^2 \tilde{q}(x,n)}{\partial x \partial n} \geq 0$ if $\psi_{qq} \leq 0$.

This analysis shows that the properties of the two boundaries $\hat{q}(x,n)$ and $\tilde{q}(x,n)$ are qualitatively the same in terms of $n$ and $x$. Hence, to simplify the analysis in the sequel we will disregard the presence of multiple equilibria for intermediate values of $\underline{q}$ and make as if the equilibrium with quality procuring firms would systematically prevail. Hence, setting $\overline{q}(x,n)\in\left\{\hat{q}(x,n),\tilde{q}(x,n)\right\}$ we have that a quality requirement $\underline{q}$ is strictly implementable if $\overline{q}(x,n)=\tilde{q}(x,n)\geq\underline{q}$ and weakly implementable if $\overline{q}(x,n)=\hat{q}(x,n)\geq\underline{q}$.

When all potential suppliers are ready to abide the minimum quality requirement $\underline{q}$, the most efficient firm wins the auction at a price $b_w=\theta''(n)+C(\underline{q})$. In this case the expected profit for any future auction which is relevant for the decision on quality procurement is

$$E\left[\Pi \mid q\geq\underline{q}\right]=\frac{E\left[\Delta\theta(n)\right]}{n}$$

where $E\left[\Delta\theta(n)\right]$ is the informational rent that firms can expect when winning any of the future auctions and $1/n$ is the probability of being the most efficient firm out of the $n$ eligible firms. As in any repeated game, the framework we are discussing is characterized by multiple equilibria with different levels of procured quality. Hence, there exists another equilibrium where the most efficient firm wins the auction, offers the required quality $\underline{q}$, but claims a lower price $b_w=\theta''(n)+C(0)$

because all rival firms do not abide the quality requirement. In this case the expected rent from any auction is smaller and equal to

$$E\left[\Pi \mid q \geq \underline{q}\right] = \frac{1}{n}\left[E\left[\Delta\theta(n)\right] - \psi(q)\frac{1-\delta^{x}}{1-\delta}\right].$$

However, independently of the particular type of equilibrium, with uncontractible quality firms may be ready to provide quality in return of future profits so that if expected profits are small then the implementable quality is also small. It then follows that if the buyer sets a too high quality requirement, the winning firms will prefer to save on quality costs and shirk on quality.[19] Hence, setting condition (1) as an equality determines a maximal implementable quality $\overline{q}(x,n)$ whose exact value depends on the particular firm's profit $E\left[\Pi \mid q \geq \underline{q}\right]$ and, in any event, the larger is the pool of eligible firms, the smaller is the probability to be the winning firm and the smaller is the expected cost difference between the most and the second most efficient firm, i.e. the informative rent $E\left[\Delta\theta(n)\right]$ is small. Note that this is also true if firms' heterogeneity is small for the properties of the distribution $f(\cdot)$, thus independently of the dimension $n$ of the pool. In addition, the longer is the contract length $x$ the smaller is the maximal implementable quality. The cause of this effect does not rely on the dimension of future expected profits, rather on the firms' possibility to retard punishment for quality shirking when $x$ is large. In the limit, if the buyer sets a once-and-for-all contract (i.e. $x = \infty$) the unique implementable quality is the nil quality because $\overline{q} = 0$.[20]

Although Lemma 1 clearly illustrates the negative effect of $x$ and $n$ on implementable quality, $x$ and $n$ have also other effects on the buyer's payoff that we now illustrate. First, a longer contract determines smaller costs for organizing the auctions which become less frequent (or a better management of economies of scale in procurement). On the other hand, a larger $x$ also implies that along the contract the buyer is stuck with a firm that may no longer be the most efficient firm in the pool of $n$ eligible suppliers. Second, a larger pool of $n$ firms implies that the price asked by the most efficient firm $b_w = \theta''(n) + C(q)$ decreases because

$\theta''(n)$ is a decreasing function of $n$. Hence, clearly if the buyer could directly control quality, that is if quality could be contractible, she would set $n$ at its largest value $n = N$ and the contract length at $\tilde{x}$ where

$$\tilde{x} \equiv \begin{cases} 1 & \text{if } K \leq E[\theta] - E\left[\theta''(n)\right] \\ \infty & \text{otherwise.} \end{cases}$$

In fact, after the first period of procurement, the buyer has to compare the cost $K$ of organizing a new auction (or non optimizing over the economies of scale) with the expected efficiency gain $E[\theta] - E\left[\theta''(n)\right]$ that can be obtained if she discards the current supplier that has expected cost $E[\theta] + \psi(q)$ for the next period and reverts to the most efficient firm that will claim a cost $\theta''(n) + \psi(q)$ for the first period.

Coming back to non-contractible quality, it is also important to notice that if the cost of restricting $n$ and or $x$ is too large, the buyer may then prefer a nil quality and in this case she would then set $n$ and $x$ exactly as she would do were quality contractible, i.e. again $n = N$ and $x = \tilde{x}$. However, when quality is valuable then at the optimum the buyer may be induced to restrict competition through $n$ and reduce contract duration $x$ taking into account that these decisions help relaxing the constraint on implementable quality $\overline{q}(x,n)$. Hence, the buyer would accept the cost of reducing $n$ and $x$ so as to increase $\overline{q}(x,n)$ only if by so doing she is interested to increase $\underline{q}$ and the actual procured quality $q$. A strictly larger than zero minimum quality requirement $\underline{q}$ is then desirable if the (net) per period value of quality $V(q) - \psi(q)$ is sufficiently large at least for certain values of $q$.

**Proposition 1.** *Optimal Procurement with Competing Firms* If the (marginal) value of quality is large, then the buyer optimally sets $n$ and $x$ such that $\overline{q}(x,n) > 0$: comparing with contractible quality or with

$q = 0$, the buyer optimally restricts the pool of eligible firms $n < N$ and the contract duration $x \leq \tilde{x}$.

**Proof. Step 1.** First recall from the proof of Lemma 1 that if the most efficient firm prefers to meet the minimum quality standard, then none of the less efficient firm is able to profitably undercut the winning offer by planning to cheat on procured quality.

The buyer's optimization program $\left(P_c\right)$ then consists in maximizing

$$S\left(\underline{q}, x, n\right) \equiv \frac{1}{1-\delta}\left[V(\underline{q}) - \psi\left(\underline{q}\right)\right] - \frac{E\left[\theta''\left(n\right)\right]}{1-\delta^{x}} - \frac{\delta\left(1-\delta^{x-1}\right)E[\theta]}{\left(1-\delta^{x}\right)\left(1-\delta\right)} - \frac{1}{1-\delta^{x}}K$$

with respect to $n$, $x$ and $\underline{q}$ subject to the implementability constraint $\overline{q}\left(x,n\right) \geq \underline{q}$.

We now show that for any maximal implementable quality $\overline{q}\left(x,n\right)$, the buyer always prefers to have the quality-implementability constraint binding, i.e. $\overline{q}\left(x,n\right) = \underline{q}$. Indeed, suppose to the contrary that $\overline{q}\left(x,n\right) > \underline{q}$, it is immediate that she can increase $n$ (and possibly also $x$) thus paying a lower price $b_w$ still obtaining the same level of procured quality $\underline{q}$. Hence, increasing $n$ (and $x$) is certainly optimal up to the point where $\overline{q}\left(x,n\right) = \underline{q}$. Furthermore, if actual procured quality is $q = 0$, then it must be that $n$ and $x$ are such that $\overline{q}\left(x,n\right) = 0$, because otherwise the buyer can increase $n$ (and possibly also $x$) thus reducing the price for procurement with no effect on quality. In other terms, it is impossible that an equilibrium can emerge where, even if the buyer sets a strictly positive minimum quality requirement $\underline{q} > 0$, still all the firms offer nil quality because in this case the buyer would deviate setting $\underline{q} = 0$ obtaining the same level of quality but at a smaller price.

Step 2. From step1, for any level of $n$, $x$ and $\underline{q}$, the implemented quality $q$ is $q = \underline{q} = \overline{q}(x,n) \geq 0$ and the program becomes

$$\max_{\{x,n\}} \; S\left(\overline{q}(x,n), x, n\right)$$

Differentiating we obtain

$$\frac{\partial S\left(\overline{q}(x,n),x,n\right)}{\partial n} = \frac{1}{1-\delta}\left(V_q - \psi_q\right)\frac{\partial \overline{q}}{\partial n} - \frac{1}{1-\delta^x}\frac{\partial E\left[\theta''(n)\right]}{\partial n}$$

$$\frac{\partial S\left(\overline{q}(x,n),x,n\right)}{\partial x} = \frac{1}{1-\delta}\left(V_q - \psi_q\right)\frac{\partial \overline{q}}{\partial x} - \log(\delta)\frac{\delta^x}{(1-\delta^x)^2}\left\{E\left[\theta''(n)\right] - E[\theta] + K\right\}$$

where $\frac{\partial \overline{q}}{\partial n} \leq 0$, $\frac{\partial \overline{q}}{\partial x} \leq 0$ and $\frac{\partial E\left[\theta''(n)\right]}{\partial n} \leq 0$. From which we can see that if $K \leq E[\theta] - E\left[\theta''(n)\right]$ then the optimal contract length is $x = 1$. If $V_q - \psi_q$ is small, then the optimal $n$ is large as well as the optimal $x$ if $E\left[\theta''(n)\right] + K > E[\theta]$. On the other hand, if $V_q - \psi_q$ is large then the optimal $n$ and $x$ are such that $n < N$ and $x \leq \overset{o}{x}$.

Reducing firms' profits, competition in terms of a large $n$ also reduces incentives to maintain their commitment for reputation by complying with the buyer's quality requirement. In addition, the buyer has another instrument to improve quality, namely contract length $x$, so that when quality is a real concern, she may want to increase quality by reducing the length of the contract thus increasing the frequency of auctions. As we have discussed above, this is not without costs for running auctions more frequently is costly both in terms of organizing auctions and in terms of lost economies of scale, but it also has the advantage of avoiding to end up stuck with inefficient firms for long periods. Hence, the Proposition shows that when non-contractible quality matters, the buyer prefers excluding some firms thus reducing competition and also possibly limiting the length of the procurement contract, as compared with what she would prefer were quality contractible or were a nil quality preferable.[21] In fact, it is only by restricting $n$ and $x$ that she can obtain a strictly positive maximal implementable quality $\bar{q}(x,n)$ and then also set a strictly positive minimum quality requirement $\underline{q}$.

An interesting corollary of the previous proposition is the following.

**Corollary 1.** Contrary to the case of fully-contractible quality, if the buyer wants to implement a strictly positive non-contractible quality, then the optimal $n$ and $x$ are not independent. If $\psi$ is (weakly) concave, then $n$ and $x$ are complement: the buyer's (marginal) benefit of $n$ increases with $x$ and viceversa.

**Proof of Corollary 1.** We are here interested in analyzing the relationship between optimal $x$ and $n$. Differentiating with respect to $n$ the first order condition for $x$, $\frac{\partial S(\bar{q}(x,n),x,n)}{\partial x}=0$ we obtain $\frac{\partial x}{\partial n}=-\frac{\partial^2 S(\bar{q}(x,n),x,n)}{\partial x\partial n}/\frac{\partial^2 S(\bar{q}(x,n),x,n)}{\partial x\partial x}$ where $\frac{\partial^2 S(\bar{q}(x,n),x,n)}{\partial x\partial x}\leq 0$ necessary for the second order condition. The expression $\frac{\partial^2 S(\bar{q}(x,n),x,n)}{\partial x\partial n}$ can be decomposed as follows

$$\frac{\partial^2 S\left(\bar{q}(x,n),x,n\right)}{\partial x\partial n}=\frac{1}{1-\delta}\left[V_q(\bar{q}(x,n))-\psi_q\left(\bar{q}(x,n)\right)\right]\frac{\partial^2\bar{q}(x,n)}{\partial x\partial n}+$$
$$+\frac{1}{1-\delta}\left[V_{qq}(\bar{q}(x,n))-\psi_{qq}\left(\bar{q}(x,n)\right)\right]\frac{\partial\bar{q}(x,n)}{\partial n}\frac{\partial\bar{q}(x,n)}{\partial x}$$
$$-\frac{\delta^x\log(\delta)}{\left(1-\delta^x\right)^2}\frac{\partial E\left[\theta''(n)\right]}{\partial n}$$

where $\frac{\partial^2\bar{q}(x,n)}{\partial x\partial n}\geq 0$ if $\psi$ is concave. as shown in the Lemma 1. Then, if $V_q-\psi_q$ is large enough and $\psi\leq 0$, then $\frac{\partial^2 S(\bar{q}(x,n),x,n)}{\partial x\partial n}\geq 0$ and $\frac{\partial x}{\partial n}\geq 0$, i.e. $n$ and $x$ are complements $\frac{\partial x}{\partial n}\geq 0$, otherwise, if the second and third term in $\frac{\partial^2 S(\bar{q}(x,n),x,n)}{\partial x\partial n}$ may prevail, so that $\frac{\partial^2 S(\bar{q}(x,n),x,n)}{\partial x\partial n}\leq 0$ and $\frac{\partial x}{\partial n}\leq 0$.

If the buyer could directly control quality or when she prefers a nil non-contractible quality, then the optimal contract length is simply $\overset{0}{x}$ either dictated by fixed-costs or by the possibility of limiting the lock–in effect of a long contract. In any case, it is independent of $n$. Interestingly, the Corollary 1 shows that this independence property of the optimal $x$ and $n$ breaks down when quality is a concern and the buyer prefers to obtain a strictly positive quality. In this case the two instruments can be both complements or substitutes. If quality is sufficiently important for the buyer and quality cost is concave, they turn out to be complements so that any event which causes an optimal reduction of firms admitted at any auction also fosters a reduction of the contract length and *viceversa*. The result in the previous Corollary thus sheds some light on a relationship which is often neglected in the literature, namely that between competition and contract length in providing rents when the buyer needs to induce the suppliers to provide non-contractible quality.

The results in Proposition 1 have emphasized a trade-off between non-contractible quality, competition and frequency of auctions. However, it is well known that in a context with repeated competition, reducing the number of competing firms and increasing the frequency of interactions are (among) the most effective conditions that foster and strengthen collusion between potential suppliers. It is important also to realize that collusion in procurement is far from being a simple theoretical curiosity and, on the contrary, it is a pervasive phenomenon which has been identified in several cases. Hence, our discussion may point to a possibly much more disappointing trade-off, namely that between non-contractible quality and collusion: if the buyer really cares for quality and cannot explicitly contract on it, she may be forced to accept collusion among suppliers.

## QUALITY AND COLLUSION

To account for the possibility that firms may collude we let firms decide whether to collude in the auction supergame or not at date $t=-1$. If a collusive agreement is reached, the most efficient firm is awarded the contract, procures the good for $x$ periods and receives the payment by the buyer which the firm sets at the highest admissible price, i.e. the reservation price $r$. All the other firms abstain from bidding or submit not acceptable / winning bids so that collusion takes place with bid rotation. Deviation from a collusive agreement is punished in the harshest way. If a defection is observed, firms compete forever in all the following auctions (i.e. we employ grim trigger strategy). All the other features of the game are unchanged and described in Section 2.[22]

Consider the optimal procurement strategies described in the previous section, including the maximum implementable quality $\overline{q}(x,n)$ and exclusion rule $\sigma$ discussed therein. We now study firms' incentives to collude under these strategies. For the collusive agreement to be sustainable at any auction stage, the second most efficient firm with cost $\theta''$ is the one with the highest incentives to deviate and should prefer not to undercut the most efficient one. If it does not deviate from the collusive agreement, this firm as well as any firm other than the most efficient one, can expect a collusive payoff that we will indicate with $\Pi^C$. If it does deviate, then it has now two possible actions: either deviating from the cartel but abiding to the minimum quality requirement, or deviate and also cheat on quality. We now investigate this firm's decision.

Each firm's efficiency is drawn anew at any auction stage so that, contrary to standard models of collusion, incentives to deviate are not fixed once and for all but depend on the period-per-period realization of efficiency. Hence, we need to consider a more sophisticated collusive agreement that also contemplates temporary phases of competitive pricing when firms' costs make too strong incentives to deviation. More precisely, colluding firms observing the realization of costs know that with a collusive pricing $r$ the second most efficient firm may bee too prone to deviation and, to avoid a break down of the cartel, the agreement prescribes a temporary reversion to competition, until in subsequent auction stages costs allow to sustain high collusive pricing. Clearly, all this does not affect firms' ability to detect deviations when costs status would concede collusive pricing and to punish such deviation.

The expected future collusive profit $\Pi^C$ for any firm is

$$\Pi^C = \frac{\delta^x}{n(1-\delta^x)}\left\{E_\theta\left[r-\theta'(n)-C(\underline{q})\right]\Pr(b_w=r)+E_\theta\left[\theta''(n)-\theta'(n)\right]\left[1-\Pr(b_w=r)\right]\right\}=$$

$$=\frac{\delta^x}{n(1-\delta^x)}\left\{\left[\overline{\theta}-E[\theta'(n)]\right]\Pr(b_w=r)+E\left[\Delta\theta(n)\right]\left[1-\Pr(b_w=r)\right]\right\}$$

where $\Pr(b_w=r)$ represents the probability of a cost realization that allows collusive pricing and $\left[1-\Pr(b_w=r)\right]$ is the probability associated to competitive pricing. Note that we are assuming that colluding firms agree to comply with the quality requirement and in the following we will study whether this is indeed the case or not.[23]

By deviating, the second most efficient firm (the one with the most incentives to deviate) can ask a price slightly smaller than $r$ thus winning the auction. This firm may decide to deviate from the collusive agreement but to comply with quality so that it obtains the following payoff,

$$\overline{\theta}-\theta''(n)+E\left[\Delta\theta(n)\right]\frac{\delta^x}{n(1-\delta^x)}$$

where again $E\left[\Delta\theta(n)\right]\delta^x/n(1-\delta^x)$ is the expected payoff for future auctions upon deviation. Alternatively, the deviating firm can also decide to shirk on quality so that its profit turns out to be

$$\overline{\theta}-\theta''(n)+\psi(\underline{q})\frac{1-\delta^x}{1-\delta}$$

Hence, the second most efficient firm at any auction does not deviate if the following incentive compatibility constraint is satisfied,[24]

$$\Pi^C \geq \overline{\theta}-\theta''(n)+\max\left\{\frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)},\psi(\underline{q})\frac{1-\delta^x}{1-\delta}\right\} \tag{4}$$

where the right hand side shows that the smaller is $\theta''$ the more profitable is the deviation for this firm and less probable is that collusion can take place.

For given procurement rules ($r, x, n$) and exclusion rule $\sigma$, (4) may or may not be satisfied depending on the value of $\theta''$. To avoid that the cartel breaks down, an optimal collusive agreement has to specify what firms should do when the value of $\theta''$ is small and does not satisfy (4). To this end, consider the boundary $\hat{\theta}$ which is implicitly defined by condition (4) written as an equality (with $\Pr(b_w = r) = \Pr(\theta'' \geq \hat{\theta})$), i.e. the value of $\theta''(n)$ which makes the second most efficient firm indifferent from deviating or not from the cartel. When firms collude, they agree to do refrain from competition at the bidding phase only when $\theta'' \geq \hat{\theta}$ and, on the contrary, when firms observe that the second most efficient has indeed low costs $\theta'' < \hat{\theta}$, they know that the collusive agreement would induce that firm to deviate and then prefer to temporarily revert to competitive pricing (till the next auction stage when $\theta'' \geq \hat{\theta}$). In these cases, the agreement requires that firms compete so that the most efficient firm wins at a price $b_w = \theta''(n) + C(q)$, as in the previous section. However, the buyer may make this event less or more probable by varying her strategies which in turn affect the boundary $\hat{\theta}$. Furthermore, consistently with the literature on collusion in stochastic environments (e.g. Rotemberg and Saloner 1986), in the following we will indicate that the buyer is able to deter collusion only if her strategy is such that there are no realizations of costs which satisfy incentive compatibility, i.e. uniquely when $\hat{\theta} \geq \overline{\theta}$. In particular, note that if the contract lasts for infinitely many periods (i.e. $x \to \infty$), or similarly, if all firms are admitted at the auction (i.e. $n \to \infty$), then the boundary $\hat{\theta}$ increases such that it can be shown to be $\hat{\theta} > \overline{\theta}$ (see the proof of Lemma 2) so that $\theta'' \geq \hat{\theta}$ is never met and collusion is certainly deterred. We can now state our first result concerning collusion.

**Lemma 2.** ***Collusion inducing*** $n$ ***and*** $x$. Reducing the contract length $x$ and /or the number of potential suppliers $n$ allows the buyer to increase implementable non-contractible quality, but at the same time it facilitates collusion among suppliers.

**Proof of Lemma 2.**

Collusion holds if the incentive compatibility constraint (4) is verified. The boundary $\hat{\theta}$ is implicitly defined by the following equality

$$\frac{\delta^x}{n(1-\delta^x)}\left\{\left[\bar{\theta}-E[\theta'(n)]\right]\Pr(\theta''\geq\hat{\theta})+E\left[\Delta\theta(n)\right]\Pr(\theta''<\hat{\theta})\right\}=$$

$$\bar{\theta}-\hat{\theta}+\max\left\{\frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)},\psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta}\right\} \tag{5}$$

where the left hand side is $\Pi^C$ and the right hand side is the profit upon deviation with or without quality provision by the deviating firm. Furthermore, $\Pi^C$ is larger the smaller is the threshold $\hat{\theta}$ because $\left[\bar{\theta}-E[\theta'(n)]\right]\geq E\left[\Delta\theta(n)\right]$.

Hence, collusion is facilitated by a smaller $\hat{\theta}$ as defined in (5). We then need to show that smaller $x$ and $n$ imply a larger implementable quality $\bar{q}(x,n)$ but also a smaller $\hat{\theta}$. Clearly, if the buyer wants to reduce $n$ and $x$, she does so in order to increase the implementable quality by relaxing the quality constraint $\underline{q}\leq\bar{q}(x,n)$. Hence, we will consider $\hat{\theta}$ defined by (5) evaluated at $\underline{q}=\bar{q}(x,n)$.

If $\max\left\{\frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)},\psi\left(\bar{q}(x,n)\right)\frac{1-\delta^x}{1-\delta}\right\}=\frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)}$ then (5) can be written as

$$\frac{\delta^x}{n(1-\delta^x)}\left\{\bar{\theta}-E[\theta''(n)]\right\}\Pr(\theta''\geq\hat{\theta})=\bar{\theta}-\hat{\theta} \tag{6}$$

A small $x$ increases the left hand side so that to preserve the equality, $\hat{\theta}$ must reduce. The effect of $n$ is similar but more complex.

On one side, a small $n$ increases the probability that a firm wins the auction, thus increasing the left hand side in (6). On the other hand, a small $n$ means that the winning firm, on average is less efficient than it could have expected to be when $n$ is large. This is captured by the fact that the term $E[\theta''(n)]$ in (6) is increasing in $n$ thus implying that a small $n$ implies a small left hand side. However, the net effect is necessarily positive. In fact, for a given path of cost realizations for any firm $i$, reducing the number of competitors makes this firm the most efficient one in the same cases as with the large $n$ plus some additional cases (in which it would not have been the most efficient firm with $n$ large). Note also that with a smaller $\hat{\theta}$, firms can secure the larger collusive rents with higher probability thus increasing expected profits.

If $\max\left\{\frac{E[\Delta\theta(n)]\delta^x}{n(1-\delta^x)}, \psi\left(\overline{q}(x,n)\right)\frac{1-\delta^x}{1-\delta}\right\} = \psi\left(\overline{q}(x,n)\right)\frac{1-\delta^x}{1-\delta}$ the effects described above are enhanced because we know that $\overline{q}(x,n)$ increases with smaller $x$ and $n$ so that, to restore the equality in (5) $\hat{\theta}$ has to further reduce.

Proposition 1 illustrates that prominence of quality for the buyer may induce her to both restrict the pool $n$ of bidders at any auction and reduce the length of contracts $x$, thus having more frequent auctions. However, we know that a small number of firms that frequently compete may be induced to collude. Indeed, Lemma 2 shows that the smaller are $n$ and $x$, the larger is the scope for and stability of collusion. This Lemma thus illustrates the seeming and disturbing trade-off between non-contractible quality implementable through reputation and the risk of inducing collusion among suppliers.

Before reaching any conclusion, however we need to check under what conditions the collusive ring prescribes to offer the required quality or not. For what we have stated above, we need to check colluding firms' incentive to provide quality both when the cost structure $\theta$ allows for collusive pricing and when it does not so, i.e. respectively when $\theta'' \geq \hat{\theta}$ and $\theta'' < \hat{\theta}$.

When $\theta'' \geq \hat{\theta}$ so that collusive pricing takes place, one needs to compare the profit for the most efficient firm with the profit this firm can obtain when it shirks on quality. The former is clearly $\overline{\theta}-\theta'(n)+\Pi^C$, and the latter is $\overline{\theta}+\psi(\underline{q})(1-\delta^x)/(1-\delta)-\theta'(n)$. With $\theta''<\hat{\theta}$ competitive pricing takes place and, by providing quality, the most efficient firm obtains a profit $\theta''(n)-\theta'(n)+\Pi^C$ and $\theta''(n)-\theta'(n)+\psi(\underline{q})(1-\delta^x)/(1-\delta)$ if it shirks on quality. Hence, in any event, the firm prefers to provide the required quality if

$$\Pi^C \geq \psi(\underline{q})\frac{1-\delta^x}{1-\delta}. \tag{7}$$

This condition allows then to compare the maximal implementable quality when firms collude with that analyzed in Section 3 and associated with competition.

**Proposition 2.** *Implementable Quality with Collusion* For given contract length $x$, number $n$ of firms admitted to the auction pool, the maximal implementable quality when firms collude is larger than with competing firms.

**Proof of Proposition 2.**

We first need to compare inequality $\Pi^C \geq \psi(\underline{q})\frac{1-\delta^x}{1-\delta}$ with the equivalent one in the case of (always) competing firm, i.e.

$$\frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)} \geq \psi(\underline{q})\frac{1-\delta^x}{1-\delta}$$

Note that $\Pi^C$ is composed by two terms weighted with different probabilities. The first component is simply $\frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)}$ and the second is $\left[\overline{\theta}-E[\theta'(n)]\right]\frac{\delta^x}{n(1-\delta^x)} \geq \frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)}$. Hence, for any $\Pr(\theta'' \geq \hat{\theta})$ we immediately have $\Pi^C \geq \frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)} (\geq \psi(\underline{q})\frac{1-\delta^x}{1-\delta})$.

Now we need to show that even if the buyer sets a $\underline{q}$ that satisfies (7), it cannot happen that the cartel organizes in a way such that the procuring firm offers nil quality asking a price $b' \equiv \overline{\theta} + C(0)$. The most efficient firm prefers to offer a nil quality if the following holds

$$\overline{\theta} - \theta'(n) \geq \overline{\theta} - \theta'(n) - \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta} + \frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)}$$

where the right hand side illustrates that if this firm deviates and offers quality, the buyer will retain that firm also for the future, whilst the cartel prescribes that each firm operates at most one period because firm are expected not to offer quality. Hence, this firm indeed offers nil quality (as prescribed by the cartel) if

$$\psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta} \geq \frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)}$$

Notice that if the buyer sets $\underline{q}$ such that

$$\frac{\delta^x}{n(1-\delta^x)}\left\{\left[\overline{\theta} - E[\theta'(n)]\right]\Pr(b_w = b') + E\left[\Delta\theta(n)\right]\left[1-\Pr(b_w = b')\right]\right\} \geq \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta} \geq \frac{E\left[\Delta\theta(n)\right]\delta}{n(1-\delta^x)}$$

indeed we may expect that a cartel can either prescribe to offer nil quality or $\underline{q}$ because

$$\frac{\delta^x}{n(1-\delta^x)}\left\{\left[\overline{\theta} - E[\theta'(n)]\right]\Pr(b_w = b') + E\left[\Delta\theta(n)\right]\left[1-\Pr(b_w = b')\right]\right\} \geq \frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)}$$

However, we also need to consider the incentives of the second most efficient firm in the nil quality providing cartel. This firm does not deviate if

$$\left[\overline{\theta} - E[\theta'(n)]\right]\Pr(b_w = b') + E\left[\Delta\theta(n)\right]\left[1-\Pr(b_w = b')\right] \geq \overline{\theta} - \theta''(n) + \max\left\{\frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)} - \psi\left(\right.\right.$$

which, for what stated above, becomes

$$\left[\overline{\theta}-E[\theta^{'}\left(n\right)]\right]\Pr(b_{w}=b^{'})+E\left[\Delta\theta\left(n\right)\right]\left[1-\Pr(b_{w}=b^{'})\right]\geq\overline{\theta}-\theta^{''}\left(n\right).$$

Being $\overline{\theta}-E[\theta^{'}\left(n\right)]\leq\overline{\theta}-\theta^{''}\left(n\right)$ and $E\left[\Delta\theta\left(n\right)\right]\leq\overline{\theta}-\theta^{''}\left(n\right)$, it is immediate that we cannot find an admissible boundary $\hat{\theta}^{'}$ that satisfies the previous conditions as an equality. Hence, if the buyer sets a quality requirement $\underline{q}$ for the cartel and $\underline{q}$ satisfies (7), then if the cartel indeed realizes, then firms will certainly offer quality $\underline{q}$.

The result is an immediate consequence of the fact that when firms collude they can expect a larger profit as compared with competition so that they are more reluctant to give up those larger (future) profits for an immediate but once and for all gain by shirking on quality. Although, in principle the implementable quality is also constrained by incentive compatibility for collusion because a larger quality requirement $\underline{q}$ increases $\hat{\theta}$ so that with a larger probability the second most efficient firm is induced to deviate from collusion. However, for any probability associated to the event of collusive pricing (i.e. $\theta^{''}\geq\hat{\theta}$), the expected profit from collusion $\Pi^{C}$ is always (weakly) larger than the expected profit with competing firms $E\left[\Delta\theta\left(n\right)\right]\delta^{x}/n(1-\delta^{x})$ and this gives the result. [25]

Proposition 2 shows that the seeming trade-off between implementable quality and collusion may well be misleading, as we now discuss in the next Section. [26]

**Optimal Procurement**

Proposition 1 shows that, if the buyer is worried about quality, she may want to restrict participation $n$ and contract length $x$. However, by Lemma 2, both these two decisions tend to induce collusion. In addition, as shown in Proposition 2, the buyer may not be necessarily impaired by collusion because this may allow implementing higher quality. At first grasp this second possibility may sound dubious because, in general, collusion makes the buyer paying a larger price for the good. On the other hand, it should be noticed that when the buyer pays a higher price, she also leaves larger rents to the contractor. Hence, there are several effect at play with collusion when quality is not contractible. Indeed, the buyer may optimally want to further restrict $n$ and/or $x$, exactly because this induces collusion, larger rents and then increases the implementable quality. We now explore how the emerging trade-off between collusion and competition is solved by the buyer.

If firms have the ability to collude, the buyer has to keep this possibility into account and adjust her strategies accordingly. Namely, she may chose a procurement contract $\left(r^s, x^s, n^s\right)$, and an exclusion rule $\sigma^s$ with a quality requirement $\underline{q}^s$, so as to systematically prevent collusion. In this case, in addition to what we have studied in the previous Section 3, the buyer must be sure that her strategies do not induce collusion, i.e. that the cartel' stability constraint (4) is not verified.[27] Alternatively, the buyer may want to set procurement rules which do induce collusion. As stated in the Introduction, this possibility can be literally taken in terms of cartel formation, or alternatively, one can interpret the collusive pact as a metaphor for (possibly lawful) agreements such as consortia among potential suppliers which are self-sustaining. With this interpretation incentive compatibility can then be seen as internal incentives for the stability of the consortium which is independent of any legal obligation among the partners in the agreement. When the buyer anticipates firms' collusion, she sets a contract $\left(r^c, x^c, n^c\right)$ and quality requirement $\underline{q}^c$ so that the cartel' stability constraint (4) is satisfied.

Even if collusion may allow the buyer to increase the implementable quality, the analysis of the optimal procurement contract in this case is even subtler than this. In fact, if the buyer's unique concern were the highest non-contractible quality, then Lemma 1 tells us that she should set $n = 1$, possibly also with a short contract (i.e. $x$ small). Clearly, with a single firm, collusion would not be an issue and the implementable quality would be (weakly) larger than with collusion associated with any number of firms $n^c$. However, this strategy of restricting $n$ to a single firm implies a large cost in terms of *inefficiency in production* because restricting the number of potential suppliers $n$, expected efficiency of the winning firm reduces. With this respect, the advantage of collusion consists in the possibility of implementing a larger quality associated with a (moderately) large number of potential suppliers $n$ which are on average more efficient than a single firm. In other terms, for any level of $n > 1$, colluding firms are ready to provide larger quality than when they compete and, at the same time, they produce more efficiently than a single firm would do. Clearly, in our model a cartel is particularly powerful in sorting out the most efficient firm also because firms are fully informed. This is clearly a strong simplification which in part can be justified on the ground that firms repeatedly participating procurement contests accumulate a lot of information on competitors and this largely eases their ability to obtain maximal rents from collusion also by increasing efficiency. On the other hand, it is also clear that even if collusion were less efficient, still the potential of improvements in productive efficiency are large as compared with a single firm. Furthermore, this potential is clearly the main motivation for the formation of consortia among suppliers.

These considerations carry in a new ingredient in our analysis, namely the buyer's concern for efficiency in production. There are several possible and non-exclusive reasons that may induce a buyer to be concerned also with efficiency. To be consistent with the objective we have stated for our buyer in the previous sections, here for example we assume that the buyer can ask for a participation fee to the sellers who are admitted in the restricted pool of $n$ firms (as in the case of "selective tendering"). We denote with $\tau^s$ the price the buyer asks for participation when the procurement contract induces competition and with $\tau^c$ when it induces collusion. Firms have a zero outside option if they are not allowed to participate, so that we can define the set of rational prices for participation with $\tau^i = \gamma\Pi^i$, $i = s, c$, where $\Pi^i$ represents the profit in the two regimes and the parameter $\gamma$ captures the fraction of firms' surplus that can be appropriated by the buyer.[28]

**Proposition 3.** *Optimal procurement*

(i) Assume the buyer does not care for efficiency in production as in the discussion so far (i.e. $\gamma = 0$). Then, the buyer prefers negotiating with a single firm when "quality is important" and she prefers running auctions with many competing firms otherwise.

(ii) If efficiency is important for the buyer (i.e. $\gamma$ is large), then the buyer prefers negotiating with a single firm when "quality is important"; she prefers running collusion-inducing auctions for intermediate value of quality and competitive auctions when quality is not important.

**Proof of Proposition 3.**

*[Sketch of the proof]*

Let us first qualify collusion-inducing procurement. The buyer induces collusion by setting $\left(r^c, x^c, n^c\right)$ and $\underline{q}^c$ so that $\overline{\theta} > \hat{\theta}^c$ where the boundary $\hat{\theta}^c$ is implicitly defined by

$$\frac{\delta^{x^c}}{n^c(1-\delta^{x^c})}\left\{\left[\overline{\theta} - E[\theta'\left(n^c\right)]\right]\Pr(\theta'' \geq \hat{\theta}^c) + E\left[\Delta\theta\left(n^c\right)\right]\Pr(\theta'' < \hat{\theta}^c)\right\} =$$

$$\overline{\theta} - \hat{\theta}^c + \max\left\{\frac{E\left[\Delta\theta\left(n^c\right)\right]\delta^{x^c}}{n^c(1-\delta^{x^c})}, \psi\left(\underline{q}^c\right)\frac{1-\delta^{x^c}}{1-\delta}\right\} \tag{8}$$

The left hand side of (8) shows that when a firm deviates, this overtakes both the other firms and the buyer. However, from the auction just after the deviation, firms recognize that a deviation occurred and adapt their strategies accordingly reverting to competition. On the contrary, the right hand side in (8) shows that the buyer sticks to her collusion-inducing strategy even if she observes a defection. Assuming stationary strategy for the buyer here simplifies the derivation of the results. In Section 6 we discuss this assumption and show that, whenever the buyer has the commitment power to leave the contractual terms and the quality requirement unchanged upon cartel defection, then she has interests to do so and thus restricts to a stationary strategy. Alternatively, if upon cartel defection she reverts to the contract and the quality requirement designed for competing firms, implementable quality with collusion would be lower but still larger than with competing firms and the proof would still go through.

(i) Assume first that $\gamma = 0$ and consider the optimal collusion-inducing contract with $n^c$, $x^c$ and associated implementable quality $\overline{q}^c$. By setting $n = 1$ and keeping the contract length at $x^c$, the maximal implementable quality becomes larger than $\overline{q}^c$ because the single firm can expect larger rents than the $n^c$ colluding firms. Not caring for efficiency, the buyer is better off by restricting $n$ to 1 if she does care for quality because she obtains a larger quality at the same price which, in both cases, is equal to the reservation price $r = \overline{\theta} + C(\overline{q}^c)$.

Furthermore, the average price paid by the buyer is smaller with collusion than in the case with $n = 1$ because the collusive ring temporarily reverts to competitive pricing smaller than $r$ when $\theta'' \leq \hat{\theta}^c$. However, when quality is indeed important for the buyer, saving on smaller prices is not what the buyer aims to obtain. Rather, she wants to implement a quality larger than that implementable both with $n > 1$ colluding firms and with competing firms and this ultimately requires a larger firm's rent and price. Hence, she prefers to contract with a single firm. It then also follows that if quality is not that important, she induces competition: with no need to extra increase of rent and quality, the buyer prefers competing to colluding firms.

(ii) Assume now $\gamma > 0$. We compare the buyer's payoff with and without collusion and when the buyer admits a single firm (i.e. $n = 1$). To simplify the analysis assume that surplus of quality is linear and positive, i.e.

$$V(q) - \psi(q) = \lambda q \geq 0.$$

The proof also holds for a strictly concave $V(q) - \psi(q)$, as long as this function reaches the (positive) maximum for a sufficiently large $q$.

The buyer avoids collusion if $\hat{\theta} > \overline{\theta}$ with $\hat{\theta}$ defined by (4) written with equality (which here does not depend on $\overline{q}$). As for the maximal implementable quality $\overline{q}(n, x)$ we consider the version for weak implementability (in multiple equilibria), so that $\overline{q}(n,x) = \hat{q}(n,x)$ is implicitly defined by

$$\frac{E\left[\Delta\theta(n)\right]\delta^x}{n(1-\delta^x)} = \psi\left(\overline{q}\right)\frac{1-\delta^x}{1-\delta}.$$

By so doing, we are clearly considering the most favorable case for implementable quality with competing firms. In this case, welfare is

$$S\left(\overline{q}(n,x),x,n\right) = \frac{1}{1-\delta}\left[\lambda\overline{q}(n,x) - \frac{\delta\left(1-\delta^{x-1}\right)}{1-\delta^x}E[\theta] - \frac{1-\delta}{1-\delta^x}K\right] - \frac{1}{1-\delta^x}\left\{\left(1-\gamma\right)E[\theta''\left(n\right)] + \gamma E[\theta'\left(\right.\right.$$

On the other hand, with a single firm $n=1$ the maximal implementable quality $\overline{q}(n=1,x)$ is defined by

$$\frac{\delta^x}{1-\delta^x}\left\{\overline{\theta} - E[\theta]\right\} = \psi\left(\overline{q}\right)\frac{1-\delta^x}{1-\delta}$$

because $E[\theta'\left(n=1\right)] = E[\theta]$ and welfare is

$$S\left(\overline{q}(n=1,x),x,1\right) = \frac{1}{1-\delta}\left[\lambda\overline{q}(n=1,x) - \frac{\delta\left(1-\delta^{x-1}\right)}{1-\delta^x}E[\theta] - \frac{1-\delta}{1-\delta^x}K\right] - \frac{1}{1-\delta^x}\left\{\left(1-\gamma\right)\overline{\theta} + \gamma E[\theta]\right\}$$

Finally, collusion holds if $\hat{\theta}^c > \overline{\theta}$ (recall that for what stated above, here $\hat{\theta}^c$ does not depend on the implementable quality without collusion) and the associated maximal implementable quality $\overline{q}^c(n,x)$ is defined by setting the right hand side equal to the left hand side in (7), i.e.

$$\frac{\delta^x}{n(1-\delta^x)}\left\{\overline{\theta}\,\Pr(\theta'' \geq \hat{\theta}^c) + E[\theta''\left(n\right)]\Pr(\theta'' < \hat{\theta}^c) - E[\theta'\left(n\right)]\right\} = \psi\left(\overline{q}^c\right)\frac{1-\delta^x}{1-\delta}$$

In this case welfare can be written as

$$S\left(\overline{q}^{c}, x, n\right) = \frac{1}{1-\delta}\left[\lambda \overline{q}^{c} - \frac{\delta\left(1-\delta^{x-1}\right)}{1-\delta^{x}} E[\theta] - \frac{1-\delta}{1-\delta^{x}} K\right] +$$

$$-\frac{1}{1-\delta^{x}}\left\{(1-\gamma)\left[\Pr(\theta'' \geq \hat{\theta}^{c})\overline{\theta} + \Pr(\theta'' < \hat{\theta}^{c}) E[\theta''(n)]\right] + \gamma E[\theta'(n)]\right\}.$$

We can now compare first the buyer's expected surplus with and without the collusive agreement. To induce collusion we know that the buyer must set a number of bidders and a contract length such that $n^{c} \leq n^{s}$, $x^{c} \leq x^{s}$. The difference in the expected surplus can then be written as

$$S\left(\overline{q}^{c}, x^{c}, n^{c}\right) - S\left(\overline{q}^{s}, x^{s}, n^{s}\right) =$$

$$\frac{1}{1-\delta}\lambda\left(\overline{q}^{c} - \overline{q}^{s}\right) +$$

$$-\frac{\delta E[\theta]}{1-\delta}\left[\frac{1-\delta^{x^{c}-1}}{1-\delta^{x^{c}}} - \frac{1-\delta^{x^{s}-1}}{1-\delta^{x^{s}}}\right] +$$

$$-K\left[\frac{1}{1-\delta^{x^{c}}} - \frac{1}{1-\delta^{x^{s}}}\right] +$$

$$-\frac{1}{1-\delta^{x^{c}}}\left\{(1-\gamma)\left[\Pr(\theta'' \geq \hat{\theta}^{c})\overline{\theta} + \Pr(\theta'' < \hat{\theta}^{c}) E[\theta''(n^{c})]\right] + \gamma E[\theta'(n^{c})]\right\} +$$

$$+\frac{1}{1-\delta^{x^{s}}}\left\{(1-\gamma) E[\theta''(n^{s})] + \gamma E[\theta'(n^{s})]\right\}$$

The positive effects of collusion are indicated by the first and the second lines. The first clearly relates to the positive effect that collusion has on quality because $\overline{q}^{c} \geq \overline{q}^{s}$ (where the inequality would be even stronger had we considered the strong form of implementability for quality with competing firms). As for the second term, recall that for any period after the first in any contract, firms do not earn any rent because they do not have private information with respect to the buyer for those periods who has to reimburse the expected cost $E[\theta]$ for any period. Hence, the longer is the contract, the larger is this reimbursement, as we have in the comparison between collusion and competition, $x^c \le x^s$.

The negative effects of collusion are represented by all the other terms. In particular, collusion requires more frequent auctions and this is costly (the third term). In addition, collusion is costly because it implies a larger price (this is effect is captured by setting $\gamma = 0$ in the last two terms) and / or it implies that producing firms are less efficient (this is described by setting $\gamma > 0$). In fact, for any $n^c \le n$ we have $E[\theta''(n)] \le E[\theta''(n^c)] \le \overline{\theta}$ and $E[\theta'(n)] \le E[\theta'(n^c)]$. Note also that the these costs of collusion are strengthened by the fact that they arise (more) frequently because $x^c \le x^s$ implies $\frac{1}{1-\delta^{x^c}} \ge \frac{1}{1-\delta^{x^s}}$.

Comparing positive and negative effects, we can immediately state that if $\lambda$ is sufficiently large (i.e. quality is important), then the first term dominates and collusion is better than competition and viceversa.

Finally, we compare collusion against negotiation with a single firm. In this latter case, the buyer sets $n = 1$ and let the implementable quality be $\overline{q}_1 = \overline{q}(n = 1, x_1)$ and the contract length $x_1$. Considering for simplicity $x^c = x_1$ we then have

$$S\left(\overline{q}^c, x^c, n^c\right) - S\left(\overline{q}_1, x^c, 1\right) = \frac{1}{1-\delta}\lambda\left(\overline{q}^c - \overline{q}_1\right) +$$

$$-\frac{1}{1-\delta^{x^c}}\left\{(1-\gamma)\left[\Pr(\theta'' \ge \hat{\theta}^c)\overline{\theta} + \Pr(\theta'' < \hat{\theta}^c)E[\theta''(n^c)]\right] + \gamma E[\theta'(n^c)] - (1-\gamma)\overline{\theta} - \gamma E[\theta]\right\}$$

The first term is negative because $\overline{q}_1 \ge \overline{q}^c$. On the contrary, the second is positive. In fact, for any $n^c$ we have $E[\theta''(n^c)] \le \overline{\theta}$ and $E[\theta'(n^c)] \le E[\theta]$. Indeed, reducing to $n = 1$ the number of firms implies both that the buyer will have to pay a larger price (on average) and that the producing firm will be less efficient, as compared with collusion and $n^c > 1$. It follows that if $\lambda$ is sufficiently large, then it is better to set $n = 1$ than having more and colluding firms and vice versa.

Finally, it follows that if $\lambda$ i very large, then contracting with a single firm is optimal, for intermediate values of $\lambda$ the best compromise between quality and efficiency is generated by collusion, if $\lambda$ is low then competition is preferable because guarantees lower prices (recall that collusion and competition are here equivalent in terms of efficiency).[29]

As we have discussed above, collusion leaves large rents to the firms and rents are necessary to induce them to provide high level of quality. However, if the buyer does not care for efficiency in production, then all what she can obtain in terms of quality with collusion can be replicated by admitting a single firm to the auction. Indeed, this guarantees the maximal rent and then the highest implementable quality. However, a single firm comes at the cost of inefficient production because the expected cost of the firm is higher than the expected cost of the most efficient firm when $n$ firms are admitted at the auction stage, i.e. $E[\theta] \geq E[\theta^{'}(n)]$. If the buyer does care for efficiency, then a trade off arise. In this case, restricting $n$ and $x$ so that collusion emerges may become optimal because it allows for the best balance balance between higher implementable quality as compared with competition and higher efficiency in production as compared with $n = 1$. Also note that when comparing optimal procurement inducing collusion with optimal procurement with competing firms, these two schemes are equivalent with respect to efficiency in production which, in both cases, is allocated to the most efficient firm. Hence, it follows that comparison between these two alternative procurement process ultimately rests uniquely on their properties in terms of implementable quality and price for procurement.

Finally we also note that if firms are homogenous as for costs so that $E\left[\theta^{''}(n)\right]$; $E\left[\theta^{'}(n)\right]$ is small independently of $n$ and as a consequence of the statistical properties of firms' efficiency, then the implementable quality with competition is also small (see condition (2)) because informational rents (profits) necessary to provide firms' incentives in quality procurement are low.

**Corollary 2.** If firms' cost heterogeneity is small, then auctioning with competing firms is dominated either by bilateral negotiation or collusion-inducing procurement.

On the other hand, if firms are very heterogeneous, then the buyer can implement a high quality also with many competing firms. Hence, collusion inducing procurement shows its maximal strength for intermediate values of heterogeneity in costs, exactly because it mediates and provides the right balance between quality with efficiency.

The analysis in this section may contribute to the literature on the mode of transaction for procurement. On the two extremes, one could seek several suppliers thus relying on the benefits of competition, or, otherwise, one could bargain with a single seller to avoid the drawbacks of competition when quality is not contractible. For example, Manelli and Vincent (1995) have analyzed sequential bargaining with take-it-or-leave-it offers designed by the buyer when non contractible quality is sufficiently important. However, it is also important to notice that the few papers that have dealt with the choice of the mode of transaction have limited the analysis mainly to a framework with no repetition. As we have previously emphasized, a main ingredient of procurement is the need to repeat the procuring process over time and, with non-contractible quality, the level of competition (i.e. the number $n$ of firms admitted at the auction) is only one relevant dimension in the procurement process. Indeed, the duration of the relationship is important. A long term relationship creates an implicit incentive so that procuring firms have incentives to establish reputation and the buyers may prefer long lasting contracts when quality is not contractible. This creates the bridge between our analysis and the important strand of literature dealing with trust and reputation formation in long-term relationships (Fehr, Brown and Falk 2004). Hence, our analysis introduces the novelty of combining these two elements in the choice of a trading procedure, the degree of competition and the length of the awarded contract.

## DISCUSSION AND EXTENSIONS

In this section we further discuss some results in the paper and consider a number of extensions of our base model checking the robustness of our previous results.

**Finite number $N$ of firms**

In the base model of the previous sections we assumed that the number of potential suppliers $N$ is infinite (Assumption 2). This has simplified the analyzed but it is certainly an extremely restrictive assumption. We aim here to show that although the equilibria in the previous sections need to be adapted tot he case of finite $N$, still the drivers of the result resist if we relax the assumption.

When the buyer induces competition between $n > 1$ firms, assume first that the firms admitted at the auction are $n < N$ and consider a candidate strategy for firms such that at any auction stage the most efficient firm provides the minimum required quality and the buyer's exclusion rule $\overset{\circ}{\sigma} = (\tilde{T}, \underline{q})$ with $\tilde{T} > 1$ contemplates the replacement of cheating firms with one among the $N - n$ firms that were previously excluded from participation. Otherwise, if the firm procures at least the minimum quality, then it is kept into the pool of qualified $n$ firms, as in the base model. We now check whether this can be an equilibrium of the game with finite $N$. Verifying if a single firm may have any incentive to deviate and cheat on quality we note that, if the firm does provide insufficient non-contractible quality, according to $\overset{\circ}{\sigma}$ the buyer will replace it with one among those $N - n$ firms excluded from the pool of potential suppliers. Now, given that all other firms will provide the required quality, the pool of active $n$ firms will remain the same for all the subsequent auction stages. Hence, the deviating firm that cheats on quality will be excluded forever, because in the candidate equilibrium at hand all other firms except the deviating one will provide minimum quality and the buyer will not need to rely on the pool of previously excluded firms. Note that for this to hold true it simply suffices that exclusion $\tilde{T}$ is at least for one auction, i.e. $\tilde{T} > 1$. This implies that as long as $n < N$ and with an exclusion rule $\overset{\circ}{\sigma}$ such that $\tilde{T} > 1$, the maximal implementable quality for the buyer is again $\overline{q}(x, n)$ as defined in Lemma 1.[30]

Importantly, note also that $n < N$ makes the threat of exclusion of cheating firms by the buyer a credible one so that the buyer has no incentive to deviate from the exclusion rule $\overset{\circ}{\sigma}$ discussed above. Indeed, at any subsequent auction all firms are identical from the buyer's view point so that replacing one firm with another in the pool of $N-n$ firms is costless. Hence, the main difference with the case $N=\infty$ is that, here, a necessary condition for $\overline{q}(x,n)>0$ is $n<N$. It is also clear that if $n=N$ with $N$ finite, there is no exclusion rule $\sigma$ that can guarantee a strictly positive maximal implementable quality.[31] It may be also possible that the coalition of all firms shirks on the quality requirement (both with and without collusion) providing nil quality. In this case, the minimal exclusion of $N-n$ auction rounds becomes also the maximal exclusion length.[22] Interestingly, this shows that a finite $N$ introduces another motive for reducing the pool of admitted firms. In fact, a smaller $n$ now makes the punishment for quality shirking tougher so that the buyer may be induced to further restrict $n$.

As previously discussed, a possible interpretation of the buyer inducing collusion can be that she allows the formation of consortia. It is the interesting to note that with finite $N$, the buyer cannot rely on consortia involving all the potential firms because, otherwise, there would be no room for punishment.

**Non-stationary strategies**

Our environment is stationary as for exogenous variables and this partially justifies our choice to analyze stationary strategies for the firms and the buyer. Concerning the case of procurement inducing collusion or cartel formation, we assumed the buyer does not change her strategy when she realizes that collusion has broken down due to a cartel deviation (see the right hand side of equation (8)). This is clearly not the unique behavior one could envisage for the buyer on this occurrence. Indeed, assume now that, on the contrary, the buyer may react when she observes a deviation. In particular, by observing a low winning bid, the buyer learns a deviation occurred and then reverts to the optimal contract for competition. This is the case when $b_w$ is such that $b_w \geq \hat{\theta}^c + C\left(\underline{q}^c\right)$

where the right hand side is the maximum price that would emerge under collusion when firms temporarily abstain from colluding. Note that if this condition is not satisfied, the buyer is not even able to detect a deviation from the cartel. We now check whether the buyer has any incentive to react differently with respect to what previously illustrated. Recall that the buyer may want to induce collusion only if this allows to increase the implementable quality. It then follows that the required quality with collusion $\underline{q}^c$ cannot be afforded by firms when they are induced to competition by a defection. Hence, when collusion breaks down, if the buyer sticks to her collusion-inducing strategies, firms will be induced to shirk on quality so that they will be excluded from future auctions. This, in turn, implies that firms' payoff following a deviation are very low and collusion is strengthened. In other words, when the buyer can commit and collusion is desirable, it is in the buyer's interest not to revert to competition in future auctions if a deviation from collusion occurs. This provides support to our choice of considering stationary strategies in the previous Sections. If, on the other hand, the buyer cannot commit to her strategies, then her sequentially optimal strategy upon (detectable) cartel defection is the optimal strategy with competition. In this case, the payoff of a firm deviating from the cartel would be larger and collusion more difficult to sustain. It is however clear that collusion can still allow for implementable quality larger than competition, albeit smaller than when the buyer can commit to her strategies. Hence, our main results qualitatively hold.

**Discontinuing procurement**

Assumption 3 requires that the buyer procures the good at any point in time. This clearly puts her in a weak position with colluding firms or when contracting with a single firm. In fact, to guarantee procurement, she has to set a reservation price $r$ which is sufficient to pay back the cost of the most inefficient firm for any level of required quality. Clearly, colluding firms can extract all the surplus by asking a price equal to $r$ and similarly the single firm within bilateral negotiations. It is then immediate that abandoning this assumption, $r$ can be optimally set at a lower value even if this may discontinue procurement for certain realization of costs. Consider now a (small) reduction of $r$ from its value in the previous analysis $r < \overline{\theta} + C(q)$. Given that the winning bid with competition is $b_w = \theta'' + C(q)$, this reduction of $r$ has no effect on firms' rents with competition and then also in implementable quality. Consider now collusion where firms are able to with a price equal to $r$. It is then clear that in this case reducing $r$ has a direct effect in limiting colluding firms' rents. We then have that the buyer pays less for procurement but implementable quality also reduces. Note however that as long as the price paid with collusion is larger than with competing firms (a necessary condition for collusion itself), our qualitative results on optimal procurement still hold.

**Scoring rules**

In principle, firms' offers could be formed by a price bid and other properties of the supply such as contractible but also non-contractible quality. Hence, in addition to what employed in the previous sections where the procurement price implicitly accounted for contractible elements in the supply relationship, the buyer could then rank offers according to a scoring rule which is a function of price, of contractible together with non-contractible quality dimensions offered by each firm. With such type of bid, the buyer may then exclude the winning firm in case the latter does not provide the promised non-contractible quality. This form of competition with bid-quality offers and scoring rule may be (although it is not necessarily the case) preferable to the one we study only if there is heterogeneity in firms' cost for quality, i.e. in case the per-period cost of firm $i$ is $\psi(q,\theta)$ with $\partial^2\psi/(\partial q\partial\theta) \neq 0$. This is an interesting line for further extending our analysis to scoring rules. [33]

**Asymmetrically informed firms**

In our analysis we have assumed that firms are fully informed (Assumption 1) which is certainly a strong assumption. The literature on repeated games with asymmetric information (see for example Compte 1998 and Kandori and Matsushima 1998) and that on collusion in repeated auctions (see Aoyagi 2003 and Blume and Heidhues 2004) have highlighted, among other results, the complexities that could arise in this context by introducing asymmetric information among bidders. This is a possible extension for our model. However, notwithstanding the intricacies of collusion in auction with asymmetrically informed bidders we expect that our simple and direct trade-offs still hold in such a sophisticated environment. Consider for example the properties of production with collusion or in a consortium. It is clear that being the members privately informed, the efficiency properties of the cartel or the consortium would be weakened. On the other hand, some information sharing among members can be expected (centralized as in many paper dealing with collusion in auctions or decentralized). It would then follow that one can still expected higher prices in collusion as compared with competing firms and at the same time larger efficiency within the cartel or the consortium as compared with contracting with a single firm.

Hence, the main drivers of our results are expected to be preserved, at least to some extent, introducing privately informed firms.

**Subjective quality evaluation**

Often important performance measure are subjective. In our contest, for example, firms could chose hence observe their investment in quality, though the realized quality observed by the buyer may be subject to noise, so that both parties have private information on what they observe. The theory of relational contracts with subjective performance measures has been developed recently by Levine (2003) and McLeod (2003) for bilateral relationships (repeated principal-agent problems). Fuchs (2006) further extend the theory among other things by discussing multiple agents situations. A common theme in these theories is that to induce the principal to report truthfully the perceived quality and to act consequently according to the prescriptions of the relational contract, the optimal contract must make the principal indifferent between reporting different performance levels of the agent. In bilateral relationships this tends to induce inefficiencies, sometimes termed "money burning": when the agent’s performance is poor and the contract prescribes a punishment for the agent, the principal cannot gain from that punishment, otherwise it would be induced to report bad performance of he agent more often.

The exclusion ("efficiency wage") strategies of the buyer we analyzed are such that the buyer does not gain from punishing a firm that did not perform as agreed, as all firms are identical and replacing one with another brings nothing to him. This means that if instead of the observable quality we would have assumed a subjective quality assessment from the buyer as described above, we would have found analogous equilibria and related results. That is, all our results can be replicated practically unchanged under the assumption of subjective quality evaluation by the buyer. The reason why no additional inefficient "money burning" is required - already pointed out in Fuchs (2006) - is that differently than in bilateral relationships, the presence of competing agents allows the principal to punish the incumbent for its poor performance without gaining anything from punishing, rather, having a competing agent to profit from it. This maintains incentives for truthful reporting.

**Persistent efficiency**

In the current framework, the buyer does not learn from auctions and contracting because in order to highlight the drivers of our results we deliberately assumed that firms' efficiency is reshuffled anew in any period (or equivalently in any auction). For sure the absence of any persistence in efficiency for firms is not very realistic and eliminates a cost that could be important for the supplier. In particular, when excluding a firm that has decided not to abide the minimum quality requirement, the buyer may the realize that although unfaithful, that firm may be very efficient and thus ready to procure at a significantly lower price than other firms in the market. In this case exclusion could be less of a scarecrow for efficient firms. We plan to extend this possibility which seems to point to a trade-off between efficiency and quality. In fact, assume that efficiency and quality are independent properties in the cost function as in the current framework. Then, less efficient firm would be conscious that they can be easily discarded and substituted and this provide the right incentives for quality provision. On the other hand, most efficient firm know that the buyer would be reluctant to discard them and are less disciplined to provide high non contractible quality.

**On cartels and consortia**

Alternatively to collusion with bid rotation as in the previous sections, firms may be able to use undetectable side transfers so that the most efficient firm wins and shares the collusive surplus with all other firms. In case the winning firm shirks on quality it will be excluded from future auctions but can be still compensated also in the future by the firms that will be allowed to participate. The number of firms that belongs to the collusive ring increases with time but collusion with quality shirking can be still an equilibrium if firms use transfers that also decline with time. Although this is a theoretical possibility, it seems to be less of practical relevance than bid rotation discussed in the previous pages (especially in the case of consortia).

Independently of the sharing mechanism, cartels or consortia may act even more efficiently than previously discussed. In fact, after the first period of contract execution, in our setup the winning firm is not necessarily the most efficient firm in the pool of colluding firm (or firms in the consortium) any more and efficiency could be boosted if production where delegated to the firm that is period-by-period the most efficient in the pool. This for example could be at least in part viable if winning firms could subcontract sub-parts of the contract. Although we expect that the main drivers of our results would be unaffected, we also plan to explore the possibility of partial cartellization where the cartel or consortium is formed by a subset of potential suppliers or where several cartels or consortia emerge and compete at the auction stage.

**Renegotiation and sub-contracting**

Although changing efficiency during the contract execution seems to point to the possibility of beneficial renegotiation, it is worth emphasizing that the trade-off between efficiency and non-contractible quality is already accounted for by the buyer (also) with the choice of contract duration $x$. Hence, a fortiori with any specific cost for renegotiation this would not take place in the current setup, unless unexpected shocks realize that are not modeled in the current framework.[34]

Sub-contracting often takes place in procurement and this may be interesting to analyze particularly with colluding firms or within consortia, as previously discussed. The point is that if in these case sub-contracting allows to increase the firms' expected rent, then also implementable quality may increase. However, the effects of sub-contracting seem to be more intricate than that because responsibility for quality provision may remain in the hands of the main contractor so that incentives for quality may well result diluted. We plan to investigate this interesting relationship between sub-contracting and non-contractible quality in procurement in a future work.

## CONCLUDING REMARKS

In this paper we have analyzed the relationships between reputation, non-contractible quality and collusion in a repeated procurement context. Repetition in the procurement relationship allow the emergence of reputation as an incentive device inducing firms to supply acceptable levels of quality. Restricting participation and contractual length, the buyer increases firms' incentives to provide quality and hence, maximal implementable quality. On the other hand, running more frequent auctions among few bidders facilitates collusive agreements among suppliers. We have analyzed this trade-off showing that when non-contractible quality and variability in suppliers' efficiency are both important, short contract duration and a collusive agreement between a few eligible sellers may maximize welfare and leave the buyer better off. We find this result interesting also because it shows that in the procurement of innovation cooperation among firms and in the limit also collusion, do not necessarily clash with innovation when R&D activity is not fully-contractible.

Finally, we also show that if quality is a major concern, the buyer can do even better by negotiating with a single firm, even if this may clash with efficiency in production. Hence, we show that the optimal procurement strategy involves a subtle balance between firms' rents, incentives for quality, collusion and efficiency.

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## NOTES

1. Reasons why some dimensions of exchanges are not explicitly contractible include complexity and prohibitive legal cost of verification; see Hart (1995) for an in depth discussion and Tirole (1999) for an evaluation of the debate on contracts incompleteness.
2. See Manelli and Vincent (1995), among others.
3. This is the case in the US and UK. See for example the 2004 US Public Procurement Guidelines. Recently, a new two-stages procedure for EU procurement has been introduced. It contemplates a pre-qualification stage where the public buyer has some discretion to exclude suppliers, followed by an auction. See EC, Directive 2004/18/EC, "On the coordination of procedures for the award of public works contracts, public supply contracts and public service contracts". For a broad discussion on procurement strategies and national legislations see Albano et al. (2006).

4. Interestingly some recent studies do confirm this intuition. For example, studying a data set for train operating companies in UK Affuso and Newbery (2002) show that (discretionary) investment is stimulated by shorter rather than longer contracts. Notwithstanding a standard hold-up problem associated with contract renewal that should point in opposite direction, the authors suggest that frequent re-procurement with short contracts disciplines suppliers who care for future re-award of the franchise.
5. On the desirability of auction when all relevant dimension of the trade relationship are contractible see Bulow and Klemperer (1996).
6. In Klein and Leffler and Shapiro firms face a perfectly elastic demand at the quality assuring price; in Allen consumers are randomly allocated among the firms charging the lowest price weakly above the "quality-assuring" one.
7. In our model, and with auctions in general, (price) signalling is impossible because the lower price is chosen by the mechanism, and the trade off quality-competition reappears.
8. Some authors have explicitly dealt with collusion in repeated auctions among asymmetrically informed firms but uniquely considering full contractibility. See for example Aoyagi (2003) and Blume and Heidhues (2004).
9. The importance of contracting timing has been recently emphasized by Guriev (2005).
10. Fehr et al. (2004) show experimentally how in a dynamic environment, when non contractible aspects become important, agents do not search for the best offer each period but rather stick to the same partner with whom they try to cooperate.
11. We will refer to buyer, auctioneer and procurer as synonymous.
12. In Section 5 we will consider the possibility that the buyer asks a participation fee to the firms that are admitted in the pool of $n \leq N$ potential suppliers.
13. Our result would be unaffected if firms could be free to choose a different quality level for any period.
14. As already discussed in the Introduction, this is certainly the case for private procurement. In public procurement this possibility may be

partially limited by national laws for public procurement.

15. For simplicity in the exposition we will treat $n$ and $x$ as continuous variables. In the sequel we will discuss when the buyer may have interest in revising her strategies along the unrolling of the game and its (limited) effect on our results.

16. Clearly, relaxing Assumption 2 in Section 6 we will consider a different optimal $\sigma$.

17. As we will discuss if a firm prefers to comply with quality it will do so forever. Furthermore, its expected profits will also depend on the behavior of other firms as for quality provision.

18. Note that $E\left[\Pi \mid q \geq \underline{q}\right]$ is here independent of $x$ because, being their efficiency $\theta_i$ not persistent, winning firms obtain an informational rent only for the first period of procurement. However, even if this were not the case so that firms are characterized by the same efficiency level $\theta_i$ for all the duration of the contract, the equivalent of (2) would be $\frac{1}{n}\frac{\delta^x}{1-\delta^x}E\left[\Delta\theta(n)\right] = \psi\left(\hat{q}\right)$ where $E\left[\Delta\theta(n)\right]$ is constant for $x$ periods cost efficiency term. Although in this case a longer contract also implies a larger rent, still we would nevertheless have $\frac{\partial\hat{q}(x,n)}{\partial x} \leq 0$ and also $\frac{\partial^2\hat{q}(x,n)}{\partial x \partial n} \geq 0$.

19. Note that equilibria with procured quality $q = 0$ exist, but these cases are necessarily characterized by $\underline{q} = 0$ so that the buyer does not expect and does not pay for quality.

20. To be precise, a quality requirement $\underline{q}$ is strictly implementable (i.e. as an unique equilibrium) if $\tilde{q}(x,n) \geq \underline{q}$ where $\tilde{q}(x,n)$ is implicitly defined by (1) as an equality with $E\left[\Pi \mid q \geq \underline{q}\right] = E\left[\Delta\theta(n)\right]/n$. Instead, $\underline{q}$ is weakly implementable (i.e. in multiple equilibria with $q = \{0, \underline{q}\}$) if $\hat{q}(x,n) \geq \underline{q}$ with $\hat{q}(x,n)(\geq \tilde{q}(x,n))$ defined using $E\left[\Pi \mid q \geq \underline{q}\right] = \left[E\left[\Delta\theta(n)\right] - \psi(q)\frac{1-\delta^x}{1-\delta}\right]/n$. Given that our results will be based on qualitative properties of implementable

quality, in the sequel we will simply deal with maximal implementable quality $\overline{q}(x,n) \in \{\hat{q}(x,n), \tilde{q}(x,n)\}$. See the proof of Lemma 1 for more details.

21. Although Manelli and Vincent (1995) consider a trade off between screening and quality implementation, by comparing an auction with all $N$ participating firms against bilateral negotiation they show that it is better to have auction (i.e. a $n = N$) if the procured good is a standardized one and bilateral bargaining (i.e. $n = 1 < N$) when the value of non contractible quality is large.

22. We briefly discuss collusion with undetectable side transfers in Section 6.

23. It is well known (Rotemberg and Saloner 1986) that fixed price collusive agreements (here at price $r$) need not be optimal in a stochastic environment. As it is customary in this literature, we will not consider more sophisticated collusive schemes in which the price smoothly varies with the realizations of costs. They are extremely difficult to derive analytically and are expected to imply similar results.

24. If a deviating firm prefers also to shirk on quality, then it does so immediately.

25. The proof also shows that, contrary to what discussed with competition, it cannot happen that if the buyer sets a quality requirement $\underline{q}$ that satisfies (7), the cartel still provide zero quality (with no claim for quality).

26. rfew.

27. As previously discussed, deterring collusion amounts setting $(r^s, x^s, n^s)$ and $\underline{q}^s$ so that the boundary $\hat{\theta}^s$ implicitly defined by (4) written as an equality (and with $\Pr(b_w = r) = \Pr(\theta'' \geq \hat{\theta}^s)$ ) is so that $\hat{\theta}^s > \overline{\theta}$.

28. Alternatively, relaxing Assumption 3 the buyer is no more constrained to set the reservation price $r$ at the highest value that guarantees certain procurement. In this case, setting a smaller reservation price may serve the buyer to appropriate part of the

suppliers' surplus which clearly increases with their efficiency.

29. Note that when the buyer decides to contract with a single firm (i.e. $n=1$), she may be able to obtain a better deal by designing an optimal screening contract. Hence, on one side the surplus discussed in the text may be a lower bound for the case $n=1$, on the other hand an optimal contract may negatively impair quality provision.

30. For sure, as in the previous sections other equilibria also exist and the same type of analysis could be then conducted.

31. Assume $n=N$ and consider $\sigma'$ such that a firm is excluded forever in case he cheats and retained otherwise (similarly for shorter exclusion). By excluding the firm the buyer limits the number of firms admitted to the next auction stage to $N-1$. However, this exclusion is not credible because the buyer gains by increasing competition with a larger number of bidders and would like to admit the firm at next auctions. Anticipating that there will be no punishment the quality provided in equilibrium will be nil. Hence, with $N=n<\infty$ any equilibrium implies nil quality.

32. A coalition-proof exclusion rule is easily obtained by complementing the initial rule with one that says that if more firms deviate, firms excluded before are reintegrated in the auction process only after all never-excluded firms have been chosen, and in order of exclusion. The punishment for multiple deviations would then be exclusion for at least $N-n$ periods.

33. The possibility to use these scoring rules may be limited by the fact that the assignment of the contract (i.e. a contractible dimension) turns out to be determined also by non-contractible dimensions. In case of public procurement, for example, this may not be viable.

34. Irrelevance of renegotiation is clearly also true if efficiency is persistent within the contract execution.

## REFERENCES

Abreu, D., (1988), "On the Theory of Infinitely Repeated Games with Discounting", *Econometrica*, 56, 383-396.

Affuso, L. and D. Newbery (2002), "The Impact of Structural and Contractual Arrangements on a Vertically Separated Railway", *The*

*Economic and Social Review*, Vol. 33, 1: 83-92.

Albano, G. L., G. Calzolari, F. Dini, E. Iossa, and G. Spagnolo, (2006), "Procurement Contracting Strategies," in *Handbook of (e-)Procurement Auction Design*, Cambridge University Press, Dimitri, Piga and Spagnolo Ed.s.

Allen, F., (1984), "Reputation and Product Quality", *RAND Journal of Economics*, 15, 311-27.

Aoyagi, M., (2003), "Bid Rotation and Collusion in Repeated Auctions", *Journal of Economic Theory*, 112, 79-105.

Bajari, P. and S. Tadelis, (2001), “Incentive Versus Transaction Costs: a Theory of Procurement Contracts”, *Rand Journal of Economics*, 32(3), 387-407.

Baker, G., Gibbons, R. and Murphy, K.J., (1994), “Subjective Performance Measures in Optimal Incentive Contracts”, *Quarterly Journal of Economics*, 109, 1125-56.

Baker, G., Gibbons, R. and Murphy, K.J., (2002), “Relational Contracts and the Theory of the Firm”, *Quarterly Journal of Economics*, 117(1), 39-84.

Bar-Isaac, H., (2003), "A partial tour around an idiosyncratic selection of models related to the interaction of firm and individual reputations," mimeo, New York University.

Bar-Isaac, H., (2005), "Imperfect Competition and Commitment" *Economics Letters*, 89(2), 167-173.

Bernheim, D. and Whinston, M., (1998), "Incomplete Contracts and Strategic Ambiguity", *American Economic Review*, 88, 902-32.

Blonski, M. and Spagnolo, G., (2003), "Relational Contracts and Property Rights", *CEPR Discussion Paper*, 3460.

Blume, A. and Heidhues, P. (2004), “Modeling Tacit Collusion in Auctions”, mimeo University of Pittsburgh and WZB Berlin.

Bull, C. (1987), "The Existence of Self-Enforcing Implicit Contracts", *Quarterly Journal of Economics*, 102(1), 147-159.

Bulow, J. and Klemperer, P., (1996), "Auctions Versus Negotiations", *American Economic Review*, 86(1), 180-94.

Che, Y.-K. and Yoo, S.-W, (2001), "Optimal Incentives for Teams", *American Economic Review, 91(3), 525-541.*

Che, Y.-K. and Gale, I., (2003), "Optimal Design of Research Contests", *American Economic Review*, 93(3) 646-671.

Compte, O., (1998), “Communication in Repeated Games with Imperfect Private Monitoring”, *Econometrica*, 66, 597-626.

Doni N. (2005), "L’affidamento mediante gara di contratti pubblici: l’importanza della reputazione" (with English summary), Politica Economica, 21(2):307-335.

EC, (2004), "On the Coordination of Procedures for the Award of Public Works Contracts, Public Supply Contracts and Public Service Contracts", Directive 2004/18/EC of the European Parliament and of the Council, March 31st,.

Ellman, M. (2006), "The Optimal Length of Contracts with Application to Outsourcing", WP Universitat Pompeu Fabra.

Fehr, E., Brown, M. and Falk, A. (2004), "Relational Contracts and the Nature of Market Interactions", *Econometrica*, 72(3), 747-780.

Fuchs, W., (2005), "Contracting with Repeated Moral Hazard and Private Evalutions", manuscript Univeristy of Chicago.

Fullerton, R. and McAfee, R.P., (1999), "Auctioning Entry into Tournaments", *Journal of Political Economy*, 107(3), 573-605.

Guriev, S. 2005, "Contracting on time", forthcoming *American Economic Review.*

Halonen, M., (2002), ”Reputation and The Allocation of Ownership”, *The Economic Journal*, 112, 539-558.

Hart, O., (1995), *Firms, Contracts and Financial Structure*, Clarendon Press, Oxford.

Hörner, J., (2002), “Reputation and Competition”, *American Economic Review*, 92(3), 644- 663.

Kandori, M. and Matsushima, H., (1998), “Private Observation, Communication and Collusion”, *Econometrica*, 66, 627-652.

Kim, I.G., (1998), “A Model of Selective Tendering: Does Bidding Competition Deter Opportunism by Contractor?”, *The Quarterly*

*Review of Economics and Finance*, 38, 907-925.

Klein, B. and Leffler, K., (1981), "The Role of Market Forces in Assuring Contractual Performance", *Journal of Political Economy*, 89, 615-41.

Kranton, K., (2003), "Competition and the Incentive to Produce High Quality", *Economica*, 70, 385–404.

Kreps, D., and Wilson, R. (1982): “Reputation and Imperfect Information,” *Journal of Economic Theory*, 27, 253-279.

Laffont, J. and Tirole, J., (1993), *A Theory of Incentives in Regulation and Procurement*, Cambridge, MA: M.I.T. Press.

Levin, J., (2003), "Relational Incentive Contracts", *American Economic Review*, 93(3), 835-847.

Macaulay, S., (1963), ”Non Contractual Relations in Business: A Preliminary Study”, *American Sociological Review*, 28, 55-67.

MacLeod, W.B. and Malcomson, J.M., (1989), "Implicit Contracts, Incentive Compatibility, and the Involuntary Unemployment", *Econometrica*, 57(2), 447-480.

MacLeod, W. B. and Malcomson, J.M., (1998), "Motivation and Markets", *American Economic Review*, 88(3), 388-411.

Manelli, A. and Vincent, D.R., (1995), "Optimal Procurement Mechanisms", *Econometrica*, 63(3), 591-620.

Milgrom, P. R. and Roberts D. J. (1982): “Predation, Reputation and Entry Deterrence,” *Journal of Economic Theory*, 27, 280-312.

Pearce, D. and Stacchetti, E., (1998), "The Interaction of Implicit and Explicit Contracts in Repeated Agency", *Games and Economic Behavior, 23(1), 75-96*

Rayo, L., (2004), ”Relational Team Incentives and Ownership”, manuscript University of Chicago.

Rotemberg, J. and Saloner, G., (1986), “A Super-game Theoretic Model of Business Cycles and Price Wars during Booms”, *American Economic Review*, 76(3), 390-407

Shapiro, C., (1983), "Premiums for High Quality Products as Returns on

Reputation", *Quarterly Journal of Economics*, 98, 659-80.

Stiglitz, J. E., (1989), “Imperfect Information in the Product Market”, in Richard Schmalensee and Robert Willig, eds., *Handbook of industrial organization*, 1. Amsterdam: Elsevier Science Publishers, 771–847.

Taylor, C., (1995), "Digging for Golden Carrots: An Analysis of Research Tournaments", *American Economic Review*, 85(4), 872-890

Tirole, J., (1999). "Incomplete Contracts: Where Do We Stand?", *Econometrica*, 67(4), 741-782.