A MODEL FOR PREDICTING THE BID DISTRIBUTION IN PUBLIC TENDERS

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ABSTRACT. According to the EU 2004/18/EC concerning the public procurement of goods and services, any Public Administration should accept the bids of all eligible suppliers in order to increase the price competition and avoid collusive actions. Building upon a past contribution to IPPC 2008, the authors have considered several tenders, analysed their bid distribution and developed a model that predicts the trend of the offers before bid opening. By using this model, a firm can determine the probabilistic shape of the bid distribution in the tender and, consequently, maximize his expected profit taking into account supply costs and probability of award.

Keywords: Public Tender, Probabilistic Shape, Expected Profit

INTRODUCTION

The process of choosing the best supplier is a very complex task both in the private and public sectors (Chen et al., 2004). In most countries, such as the EU, the public sector is regulated by a number of public procurement laws that bring legislative requirements into force.

Public and private procurement systems share the same essential purpose of finding supply sources at the cheapest price and acceptable quality, however these two systems differ. Public procurement, in fact, must follow prescribed and fixed procedures and transparency is imperative (Panayiotou et al., 2004). This rule is imposed by the need for optimizing the use of the large amount of taxpayers' money. In 2003 public procurement had a potential value of more than 1500 billions euro, approximately 16% of the EU Gross Domestic Product (Lewis, 2007). Consequently, Governments need to encourage competition among candidate suppliers, not only in terms of price but also in terms of environmental and qualitative parameters (Malmberg, 2003). The ultimate goal is to obtain the best possible public goods or services and avoid collusion. Specific laws exist in this regard.

In the European Union the main law for public tendering is the 2004/18/EC Directive, also called the Public Procurement Directive, that was enacted by the European Parliament (2004). According to this directive, a public contract should be awarded on the basis of objective criteria which ensure compliance with the principles of transparency, non-discrimination and equal treatment, and at the same time guarantee that tenders are assessed in conditions of effective competition. As a result, only the following two award criteria are allowed: the "Lowest Price" (LP) and the "Most Economically Advantageous Tender" (MEAT) criterion. Typically, the LP principle is significant when money saving is pursued. On the other hand, when the MEAT criterion is used, and depending on the goals of the contract, various parameters are considered for contract award, such as delivery or completion time, operational costs, cost-effectiveness, quality, aesthetic, functional and environmental characteristics, technical merit, after-sales service and technical assistance, etc. In the MEAT case, the contracting authorities define the multiple criteria for contract award and the relative weighting, or at least the descending order, of the importance of selected criteria.

While the LP criterion essentially takes into account the traditional purchasing concept of the lowest price for the contract, the MEAT criterion reflects the necessity for a more complex purchasing management process that takes into account other key parameters for vendor selection, in addition to price. In advertising tenders, a public authority may choose either an open or a restricted procedure. In some cases, a "negotiated" procedure is allowed also. The open procedure is typically used when the requested product is a commodity or services are relatively simple. In these cases any interested vendor is allowed to bid. When customized products or complex services are required, the restricted procedure is the best option. This approach allows a public authority to assess the

technical and economic competence of bidders on the basis of their prequalification. In the case of open procedure LP is the selection option, while the MEAT criterion can be also used with a restricted procedure.

Procurement tendering procedures have been largely addressed in the auction theory field in which four different auction modes are identified (Klemperer, 2004):

- *Ascending – Bid auction (English auction)*: buyers raise their offer from a set price until no other buyer makes any higher offer. The winner pays the highest offered amount. Offered purchase prices can be announced by the auctioneer, the buyers themselves or submitted electronically. Another similar form is the *Japanese auction*: as the auctioneer raises price, buyers no longer willing to purchase quit the auction until only one remains.

- *Descending – Bid auction (Dutch auction)*: the auctioneer starts from a set selling price which is progressively reduced until a buyer announces its willingness to purchase at the latest reduced price.

- *First Price Sealed Bid (FPSB) auction*: each buyer submits a bid independently, without knowing the bids of other buyers, and the good is sold to the buyer with the highest bid. The amount paid is that offered by the winner.

- *Second Price Sealed Bid (SPSB) auction*: this is a form similar to the previous one. It differs only for the fact that the winning bidder has to pay the amount of the second highest bid. This theoretical model, proposed by Vickrey in his seminal work on auction theory (1961), has few real applications.

Auctions generally are characterized by information asymmetry. The auctioneer does not know how much each bidder values the object of transaction. Similarly, each bidder does not know the valuation by each other bidder. The pertinent literature introduces some models that describe different situations. The Independent Private Value (IPV) model (Narahari et al., 2009) is based on the hypothesis that bidders are risk-neutral (as the auctioneer), symmetrical (value in the same way), maximize their profit and the winner's payment is a function of her offer. Each bidder knows the value v_i attributed to the transaction object, but not that of other bidders. In addition, each individual evaluation does not reflect that of other bidders. The only information available to all competitors and the auctioneer is the probability distribution $F(v_i)$ from which all the evaluations are derived. This distribution is common to all values because of the assumption of symmetry for all bidders. An alternative model is the

Common Value (CV) (Wilson, 1969). The transaction object has the same unknown value V for all competitors. Consequently, each bidder makes the evaluation v_i that results from the conditional probability distribution $F(v_i | V)$. In this case the knowledge of other bidders' evaluation gives information about the true value of the object to be transacted. The IPV model is applied to auctions where bidders are not interested in reselling what they buy (Menezes and Monteiro, 2005), as in the case of collectors of artworks. Differently, the CV model is used for modelling auctions such as those for oil deposit rights.

An intermediate case between IPV and CV is the model of Affiliated Values (AV) (Milgrom and Weber, 1982). The value v_i of the transaction object is still subjective, but it is influenced by other evaluations according to a specific relation called affiliation condition.

These models have been introduced according to the assumption that an auctioneer is the seller and bidders are the buyers that evaluate what is to be purchased. This assumption does not lose its validity in the case of procurement auctions in which an auctioneer is the purchaser and bidders are the suppliers that sustain costs in providing the required good or services. The above mentioned procurement options are the focus of our research that builds upon the applicability of the auction theory to both selling and purchasing tenders.

Our study focuses on the competitive procurement of public projects that are generally awarded according to the FPSB option. The Nash equilibrium of a FPSB selling auction under the IPV hypothesis was analyzed (McAfee and McMillan, 1987) to find out the equilibrium bidding function for the i-th bidder. The function assigns a value v_i to the transaction object and assumes that all competitors make a bid according to the same continuously increasing function $B(v_j)$ with $j \neq i$. Its expected profit is equal to

$$\pi_i = (v_i - b_i) \cdot [F(B^{-1}(b_i))]^{n-1} \qquad (1)$$

where $[F(B^{-1}(b_i))]^{n-1}$ is the probability that the value given to the transaction object by n-1 competitors is less than v_i and $F(v)$ is the common value distribution for all bidders. According to Nash's assumptions, i.e., bidders are rational and symmetric, the bidding function for all competitors can be found out by imposing the profit maximization of each bidder. These issues are still valid when procurement auctions are considered. The only difference is that the expected profit of the i-th bidder is $(b_i - c_i)$, where c_i is the bidder's cost of providing the required product or service.

According to equation (1), bids are a function of the value attributed to the transaction object. Such a formulation requires the definition of a probabilistic distribution of the costs of the public work for the bidders in order to foresee their offers. An alternative is the extrapolation of the bidding function from the empirical data of past auctions of public projects that are similar to those addressed in this paper. In this way, it is possible to determine the distribution of bids as a function of an *a priori* known input (in our case, the reserve price), without defining the parameters of the bidders' costs distribution: these contain private information and, consequently, are difficult to estimate.

In this study the authors found the best fitted distribution of bids by using the data of an observed set of public tenders. The considered set encompasses projects of the same type and price range. The resulting distribution is parametric. This curve can be used to forecast the distribution of bids for similar projects before the bidding event. Once this distribution is known, it is possible to analyze the profit maximization behaviour of a given bidder.

The paper is organized as following: in the next section the methodology for reaching the above mentioned objectives is addressed. The following section introduces the application context of the methodology, namely an Italian public agency that is in charge of the national roadway and highway stock. In this regard, a set of 19 tenders for roadway reconstruction is considered. The section "Results" shows the possibilities given by the presented methodology. Lastly, in the "Conclusion", some of the findings are discussed.

METHODOLOGY

The definition of the bid, b_i as a stochastic variable, is widely discussed in the literature concerning auction theory. Different opinions about the shape of the probability density function of this variable are found: uniform (Cauwelaert and Heynig, 1978; Fine and Hackemar, 1970; Grynier and Whittaker, 1973), normal (Cauwelaert and Heynig, 1978; Mitchell, 1977; Skitmore and Pemberton, 1994), lognormal (Weverbergh, 1982) and Weibull (Oren and Rothkopf, 1975). Some distributions, like the normal, uniform and triangular ones, best fit the case of bidders with comparable cost structures; while others, such as the Pareto and exponential ones, best describe the cases in which lower bids are more likely than larger ones (Naldi and D'Acquisto, 2008).

The basic hypothesis of this study is that bids form a Gaussian distribution $g(\mu,\sigma)$. This hypothesis is consistent with the necessary condition of being an equilibrium bidding distribution (Monteiro, 2006). A confirmation can be found in empirical observations: studies of bids distribution in past construction procurement auctions

confirm the normality hypothesis (Skitmore, 2002; Conti and Naldi, 2008).

The normal distribution $g(\mu,\sigma)$ is also verified in this study. The data from a set of 19 past bidding events are analyzed. In each event, submitted bids are considered as a sample of the population of the overall possible bids. In order to verify the validity of the normality assumption, the Jarque-Bera hypothesis test (Jarque and Bera, 1980) is applied to each component of the set. Having defined the form of bidding distribution, the second step is identifying its typical parameters: the expected value μ and the standard deviation σ. Using the available data set of homogeneous procurement auctions (projects of the same type and price range), the relation of the parameters μ and σ with the Reserve Price B is found. In fact, according to the Italian law, Reserve Price is publicly known before the submission of the offers. Consequently, this is the only public information before the opening of the sealed bids. The cited relation results from the interpolation of points $(B_i, \mu_i,)$ and (B_i, σ_i). In this regard, different interpolating functions are compared in order to choose those with the highest coefficient of determination R^2.

The obtained functions $\mu=\mu(B)$ and $\sigma=\sigma(B)$ are used to estimate these parameters *ex-ante* for an additional similar procurement tender, whose Reserve Price is B_0. The obtained bidding distribution is compared with the Gaussian that shows the mean value and standard deviation of actual bids as characteristic parameters. The fitness of *ex-ante* distribution to *ex-post* one is measured by evaluating the maximum difference (in absolute value) between the two cumulating functions.

Lastly, the perspective of the bidder i in this auction is analyzed. If she is rational, she wants to maximize her expected profit. Building upon the previously introduced formulation and replacing the probabilistic cost function $F(B^{-1}(b_i))$ with the determined cumulative normal distribution, the expected profit is equal to

$$\pi_i = [b_i - c_i] \cdot [1 - G(b_i)]^{n-1} \tag{2}$$

where b_i and c_i are respectively the requested price and the cost of carrying out the public project for the i-th bidder, and n is the number of bidders taking part to the auction and G is the cumulative Gaussian function having $\mu=\mu(B_0)$ and $\sigma=\sigma(B_0)$. Therefore $[1 - G(b_i)]^{n-1}$ is considered as the probability that the bid of $(n - 1)$ competitors is no less than b_i. A set of profit curves is found by varying n and the b_i that maximizes π_i is found for each of them.

APPLICATION OF THE METHODOLOGY

The case study has been carried out at the ANAS S.p.A Company. ANAS is the management agency of the Italian road and motorway network of national importance. Its operations are monitored and supervised by the Ministry of Transport and Infrastructure. ANAS is a public limited company owned by the Ministry of Economy and Finance.

The Company, whose share capital amounts to 2,269,892,000 Euros, is responsible of the state-owned road network that consists of 27,000 kilometres of roads and motorways. It has 6,588 employees. In addition to the 1,350 kilometres that it manages directly, the Company monitors 5,657.9 kilometres of the motorway network under concession.

The functions delegated to ANAS are the following:

1. management, ordinary and extraordinary maintenance of highways and motorways of national importance;

2. adaptation and progressive improvement of the highway and motorway network, and road markings and signs;

3. construction of new major highways and motorways, including toll routes, both directly and indirectly through external contracting firms;

4. information services for users, starting from signal equipment;

5. monitoring of works being carried out by contracting firms and control of motorway management;

6. implementation of laws and regulations concerning the management of the highways and motorways stock as well as of traffic safety and signage;

7. development of the provisions that pertain to the traffic safety on highways and motorways;

8. development of and participation in studies, research and experiments regarding road and traffic conditions.

The analysed public tenders are 19. They were awarded according to the LP criterion. Table 1 shows the data of each tender, including the Reserve Price B, the Safety Payments S (i.e., bidder's cost of safety compliance), the Mean Value μ, and the Standard Deviation σ.

TABLE 1

Collected data of 19 public tenders (in Euro)

	Base Auction B	Safety Payments S	Mean Value μ	Standard Deviation σ

1	11,500,508	533,007	10,379,036	570,057
2	34,142,984	2,128,576	30,321,419	1,896,182
3	15,775,701	0	13,883,707	590,294
4	7,079,162	598,571	6,370,908	406,390
5	32,643,645	2,153,418	32,001,876	337,074
6	15,551,867	670,039	15,108,994	504,771
7	18,232,581	629,800	17,446,161	440,516
8	22,258,649	1,105,440	22,002,461	428,983
9	36,005,007	1,355,531	35,588,803	326,217
10	14,685,000	300,000	12,066,248	714,945
11	13,969,026	923,860	12,988,425	825,286
12	8,585,214	175,208	6,608,733	566,097
13	20,167,100	516,450	18,437,957	664,427
14	28,527,313	1,328,443	23,456,672	692,483
15	174,946,951	3,529,631	78,611,031	35,397,181
16	47,981,802	430,116	35,677,411	2,460,336
17	14,900,110	768,393	10,980,563	714,921
18	6,268,379	235,800	5,442,977	295,403
19	90,078,458	1,820,287	78,347,792	3,954,216

All the above listed tenders have more than 5 bidders and pertain to construction work of similar nature whose base monetary value ranges from 6,000,000 to 175,000,000 Euros.

RESULTS

Table 2 summarises the results of the Jarque-Bera test. Only 15 out of 19 tenders have positive results (the normal distribution cannot be rejected). One is negative (the normal distribution must be rejected). The remaining 3 tenders become positive if outlier bids are not considered. In the last 3 tenders, the normality hypothesis can be accepted if we consider that some bidders deliberately overbid because they did not need the work or bid the work simply for maintaining a business relationship with the auctioneer (Skitmore, 2002).

TABLE 2

Results of the Jarque-Bera test

Tender	J-B Test	No. bidders	Tender	J-B Test	No. bidders
1	POSITIVE	15	11	POSITIVE	11
2	POSITIVE	39	12	ONE OUTLIER	52
3	POSITIVE	23	13	POSITIVE	28
4	POSITIVE	17	14	NEGATIVE	21
5	ONE OUTLIER	20	15	POSITIVE	9
6	POSITIVE	12	16	POSITIVE	7
7	ONE OUTLIER	9	17	POSITIVE	11
8	POSITIVE	20	18	POSITIVE	20
9	POSITIVE	19	19	POSITIVE	14
10	POSITIVE	19			

The next step consists of defining *a priori* the mean value and standard deviation of the *ex-ante* normal bidding distribution as a function of Reserve Price. The bids themselves are already a function of Reserve Price, according the (3):

$$b_i = (1 - b_{\%,i}) \cdot B + S \tag{3}$$

where $b_{\%,i} = \dfrac{(-b_i + S + B)}{B}$ is the percentage discount offered by the

bidder *i*. If b_i is a stochastic normal distributed variable, also $b_{\%,i}$ is the same because of its linear relation with b_i. Consequently, the interpolation of the mean value and standard deviation can be made for $b_{\%,i}$ and the result can be used for estimating b_i by means of (3). The interpolation is based on all the data of Table 1, excluding the data of Tender 2 that are used to verify the effectiveness of the proposed method. Different interpolating functions, the exponential function, power function and second degree polynomial function, are compared. The last one is the function that show the best fit to the data of Table 1, both in terms of μ and σ. The polynomial functions, showed in Figure 1, have higher R^2 values than those of the other functions.

FIGURE 1

Interpolation of μ=μ(B) and σ=σ(B)
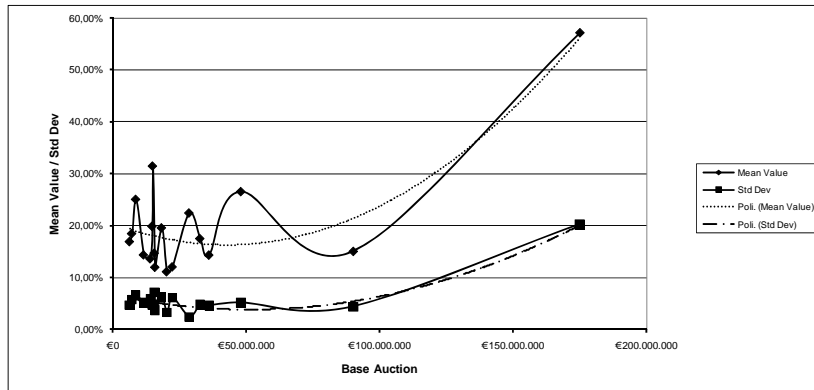
The found interpolating functions are:

$$\mu_{b\%,i} = 2.2818 \cdot 10^{-15} \cdot B_i^2 - 1.9545 \cdot 10^{-7} \cdot B_i + 2.0474 \cdot 10 \qquad (4)$$

$$\sigma_{b\%,i} = 1.0593 \cdot 10^{-15} \cdot B_i^2 - 1.0844 \cdot 10^{-7} \cdot B_i + 6.6066 \qquad (5)$$

Equations (4) and (5) are used for estimating *ex-ante* the mean value and standard deviation of the discount percentage in the bids of for the Tender 2 (previously not considered in the interpolation). Table 3 compares the forecasted and actual values of $\mu_{b\%}$ and $\sigma_{b\%}$.

TABLE 3

Forecasted vs. actual parameters for bidding distribution of Public Tender 2

Reserve Price B [€]	Forecast ed Mean Value $\mu_{b\%}$	Actual Mean Value $\hat{\mu}_{b\%}$	Forecasted Std. Dev. $\sigma_{b\%}$	Actual Std. Dev. $\hat{\sigma}_{b\%}$
34,142,984.16	16.461%	17.427%	4.139%	5.626%

Figure 2 compares the *ex-ante* and *ex-post* normal bidding distribution functions. The largest difference between the two corresponding cumulating functions $G(\mu_{b\%}, \sigma_{b\%})$ and $G(\hat{\mu}_{b\%}, \hat{\sigma}_{b\%})$ is 0.1280 (out of a possible highest value equal to 1) for a percentage discount of 20.354%, while the average error is equal to 0.0363.

FIGURE 2

Comparison of actual and forecasted bidding distribution functions
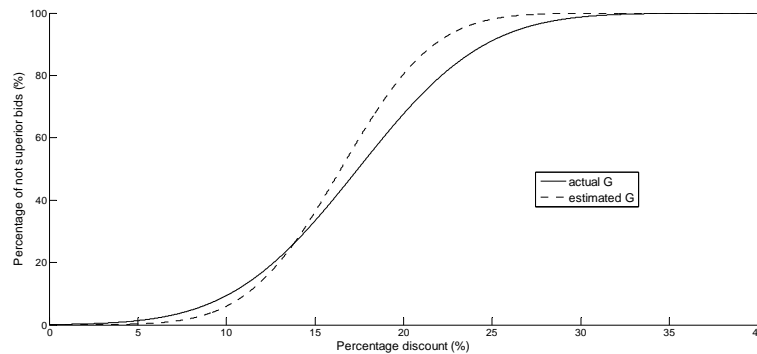
For instance if a 25% discount is considered, 91.09% of the bids will have a lower discount according the actual distribution, while in the case of forecasted distribution, this discount will be offered by 98.04% of all bids.

The forecasted distribution can be considered as a probability distribution. $G(b)$ is the probability that a given bid of Tender 2 has a value no greater than that of b. Therefore, this function is used to estimate the expected profit π_i of the i-th bidder according to equation (2). Figure 3 shows the expected profit (in Euros) depending on the magnitude of the offered discounts and number of competitors. The shape of the curves reflects the fact that very small discounts result in a very low probability of bid award initially. As the discount increases so does profit given the higher probability of bid award. Successively the increasing probability of bid award puts a limit to profit that begins decreasing. Of course the higher is the number of suppliers, the lower are the expected profits. In the considered tender study, the calculated hypothesis is equal to 20 million Euros. When the discount reaches a certain value (about 30% in Tender 2), the probability of bid award $[1 - G(b_i)]^{n-1}$ is very close to the unit, Consequently, whatever is the number of bidders, the expected profits are the same. Obviously, when the discount is such that the cost c_i is more than the bid b_i, the expected profit becomes negative.

FIGURE 3
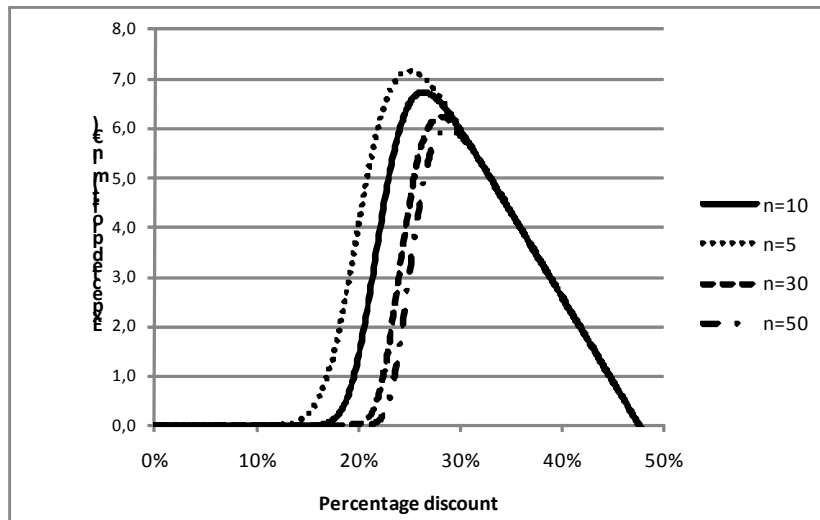
Expected profit vs. percentage discount in Tender 2

Figure 4a confirms the trend that emerges in Figure 3. The expected profit of the *i-th* bidder is decreasing with the number of bidders, but less than linearly. In the same way, the optimal discount (with the highest expected profit) is increasing less than linearly with the number of bidders, as shown in Figure 4b.

FIGURE 4

Expected profit vs. number of bidders (a) and optimal discount vs. number of bidders (b) in Tender 2
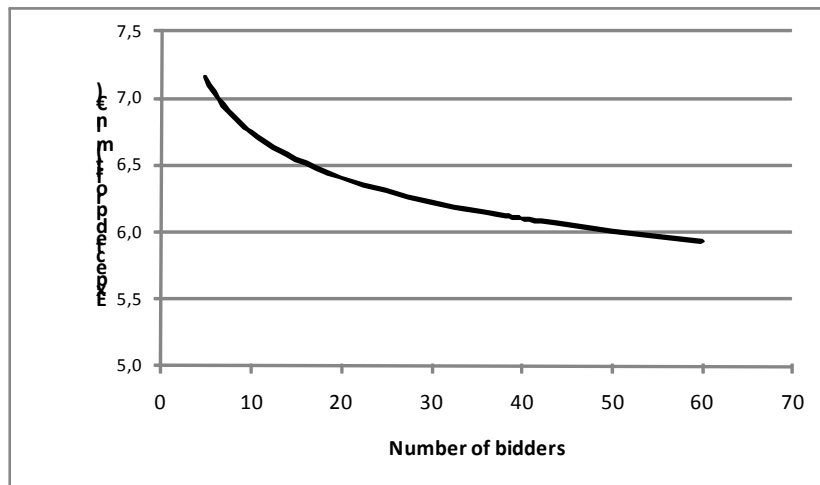
(a)
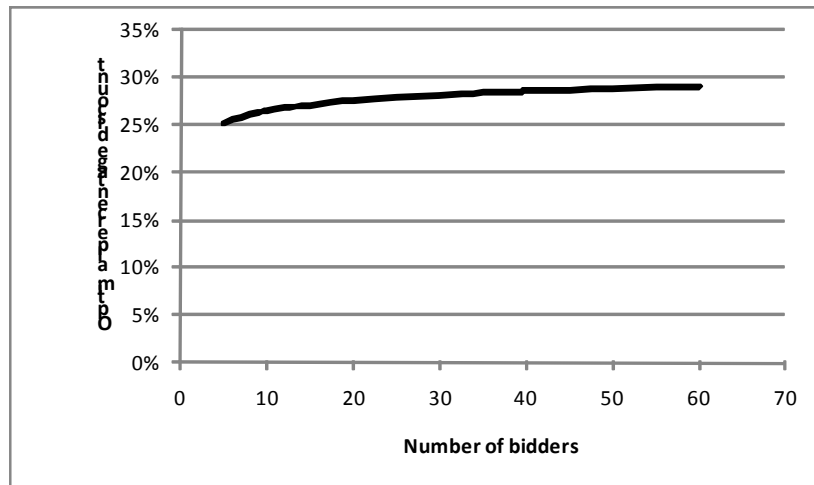
(b)

If the hypothetical *i*-th bidder had a rational behavior, she would have offered the discount that maximizes her expected profit, equal to 28.5% This corresponds to n=39 bidders (as shown in Figure 4b): only one of the offers is better than hers.

CONCLUSIONS

The analysis of actual bids in such a procurement auction offers useful opportunities to both the auctioneer (a Government or public firm such as ANAS) and bidders.

Figure 4b shows that the increase of bidders determines the growth of the optimal discount and consequent benefit for the auctioneer. The marginal benefit of a larger number of bidders decreases: for instance, it is more advantageous for ANAS to have 6 instead of 5 bidders than to have 40 instead of 39 bidders. At the same time, the auctioneer has an additional administrative cost (Costantino et al., 2008) that grows linearly according to the number of bidders. Consequently, ANAS can find the optimal number of bidders by equalizing marginal benefit to marginal administrative cost. In order to find the relationship between optimal discount and number of bidders, it is necessary to estimate the winner bidder's cost of carrying out the work. Due to the very high incidence of the awarding price in comparison with the additional administrative cost, the optimal number of bidders is expected to be high in this kind of procurement auctions. Otherwise, this number is lower in the public tenders of projects of smaller monetary value. Current Italian laws do not allow the public auctioneer to define the exact number of bidders, but public firms and governments can limit the number of invited firms with the "negotiated" procedure.

Firms can use this model for estimating their optimal bid in future auction events (which encompass the same type of projects with a comparable reserve price) without developing a function for estimating the cost of supplying the work that is incurred by other competitors. The only input required for this model, but not known by the bidder before submitting the tender, is the number of competitors. The further development of this model should focus on the probabilistic definition of the number of bidders participating in the procurement auction.

The presented model is based on a limited sample of public tenders. Actual data of project of the same type and reserve price are not easy to find, future research should address a larger sample of analyzed tenders in order to better support the findings of the presented study.

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REFERENCES

Cauwelaert F.V., Heynig E. (1978) "Correction of bidding errors; the Belgian solution", *Journal of the Construction Division*, American Society of Civil Engineers; 105(CO1):13–23.

Chen C.-C., Yeh T.-M., Yang C.-C., (2004). "Customer-focused rating system of supplier quality performance", *Journal of Manufacturing Technology Management*, 15(7): 599-606.

Conti P.L., Naldi M. (2008) "Detection of anomalous bids in procurement auctions", *Decision Support Systems*, 46: 420-428

Fine B., Hackemar G. (1970) "Estimating and bidding strategy", *Building Technology and Management*, 8–9.

Grinyer P.H., Whittaker J.D., (1973) Managerial judgement in a competitive bidding model. *Operational Research Quarterly*, 24(2):181–191

Jarque C.M., Bera A.K., (1980). "Efficient test for normality, homoschedasticity and serial independence of regression residuals". *Economics Letters*, 6: 255-259.

Klemperer P., (2004). "*Auctions: Theory and Practice*", Princeton University Press.

Lewis H., (2007). "*Bids, Tenders & Proposals: Winning Business Through Best Practice*", Kogan Page

Malmberg F., (2003). "Competitiveness in the context of procurement", in Atkin B., Borgbrant J., Josephson P.E., *Construction process improvement*, Blackwell Publishing.

McAfee R.P., McMillan J., (1987). "Auction and Biddings", *Journal of Economic Literature*, 25(2): 699-738.

Menezes F.M., Monteiro P.K., (2005). "*An Introduction to Auction Theory*", Oxford University Press.

Milgrom P.R., Weber P.J., (1982). "A theory of auction and competitive bidding." *Econometrica*, 50: 1089-1122.

Mitchell M.S. (1977) "The probability of being the lowest bidder" *Applied Statistics*; 2(2):191–194.

Monteiro P.K., (2006). "The set of equilibria of first-price auction", *Journal of Mathematical Economics*, 42: 364-372.

Myerson, R. B., (1981). "Optimal Auction Design" *Mathematics of Operations Research*, 6: 58 -73.

Naldi M., D'Acquisto G. (2008). "Performance of the Vickrey auction for digital goods under various bid distributions", *Performance Evaluation* 65(1): 10 - 31

Narahari Y., Garg D., Narayanam R., Prakash H., (2009). "*Game Theoretic Problems in Network Economics and Mechanism Design Solutions*", Springer.

Oren S.S., Rothkopf M.H. (1975) "Optimal bidding in sequential auctions". *Operations Research*, 23(6):1080–90.

Panayiotou, N. A., Gayialis, S. P., Tatsiopoulos, I.P. (2004). "An e-Procurement System for Governmental Purchasing." *International Journal of Production Economics*, *90*: 79–102.

Riley J.G., Samuelson W.F., (1981). "Optimal Auctions." *American Economic Review*, 71: 381-392.

Skitmore, M. (2002) "Identifying non-competitive bids in construction contract auctions", *Omega*, 30: 443 – 449

Skitmore RM, Pemberton J. (1994) "A multivariate approach to contract bidding mark-up strategies", *Journal of the Operations Research Society*; 45(11):1263–1272

Vickrey W., (1961). "Counterspeculation, Auctions and Competitive Sealed Tenders", *Journal of Finance*, 16(1): 8-37.

Weverbergh M., (1982) "Competitive bidding: estimating the joint distribution of bids". *Working paper 82-79, December,* Centrum voor Bedrijfseconomie en Bedrijfseconometrie Universiteit Antwerpen—USFIA.